Massive separation of turbulent Couette flow in a one-sided expansion channel

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Abstract
Direct numerical simulation has been performed to study wall-driven flow over a backward-facing step at Reynolds number \( Re = 5200 \) based on the step height \( h \) and the upper-wall velocity \( U_w \). The flow configuration consisted of a step with height equal to that of the upstream channel yielding an expansion ratio 2:1. Instantaneous enstrophy contours revealed the formation of Kelvin–Helmholtz instabilities downstream of the step. Intense velocity and vorticity fluctuations were generated in the shear-layer formed between the bulk flow and the massive recirculation zone in the lee of the step. Extraordinarily high turbulence levels persisted in the center region even 7.5h downstream of the step, i.e. where the separated shear-layer reattached to the wall. A fully redeveloped Couette flow cannot be reached in the downstream part of the channel due to the principle of mass conservation. The local wall pressure coefficient gave evidence of an adverse pressure gradient in the recovery region, where a Couette–Poiseuille flow type prevailed.

1. Introduction

Turbulent flow over a backward-facing step (BFS) is a simplified case of the general family of separated flows with widespread industrial applications. Although its geometry is simple, the flow physics is still complex. Typical prototypes of BFS flows are the boundary layer, the plane channel and the Couette flow cases, see e.g. Eaton and Johnston (1981). A common feature of these flows is the existence of a shear-layer emanating from the step corner and reattaching further downstream leading to the formation of a recirculation bubble. The presence of the internal shear-layer and the massive recirculation zone gives rise to complex flow dynamics which for instance affect the turbulence production and Reynolds stress anisotropy.

The understanding of the flow over a backward-facing step was initially acquired by experiments and two-dimensional numerical simulations. The early studies were performed by Abbot and Kline (1962) and Goldstein et al. (1970). This type of flow is characterized by the channel expansion ratio \( ER \) and the upstream Reynolds number. Armaly et al. (1983) conducted experiments on air flow over a backward-facing step with an expansion ratio of 2:1 and provided information on the relationship between the Reynolds number and the reattachment length \( X_r \). The authors covered a wide range of Reynolds numbers from about 50 to 6000 and found that \( X_r \) tends to increase with the increase of \( Re \) in the laminar flow regime and decrease in the transitional one while \( X_r \) remains relatively constant in the fully-developed turbulent state. The findings of Kuehn (1980), Durst and Tropea (1981), Ötügen (1991), and Ra and Chang (1990), on the other hand, showed that the reattachment length increases with the expansion ratio.

Owing to the rapid developments in high-performance computing, three-dimensional numerical simulations of turbulent flow over a backward-facing-step have been performed since the late 1980s. Friedrich and Arnal (1990) studied high Reynolds number turbulent backward-facing step flow using the large-eddy simulation (LES) technique. This technique was also used by Neto et al. (1993) who performed a numerical investigation of the coherent vortices in turbulence behind a backward-facing step. Later, Le et al. (1997) provided an extensive DNS study of turbulent boundary layer flow over a backward-facing step with an expansion ratio of 6:5 and reported a reattachment length equal to six step heights.

Since the BFS problem is not homogeneous in the streamwise direction, proper inflow conditions are to be employed in order to provide a realistic fully turbulent flow at the input. Several methods of different complexity have been used by researchers in the past years to generate suitable inflow conditions. In the boundary layer backward-facing step flow, Le et al. (1997) imposed a mean velocity profile for a flat plate turbulent boundary at the input (Spalart, 1988), whereas Meri and Wengle (2002) utilized a time-dependent inflow condition from a precursor Poiseuille flow simulation to perform DNS and LES of pressure-driven backward-facing step flow. A recent DNS study on the same flow problem was performed by Barri et al. (2010), where the authors used a cost-effective method to generate realistic dynamic inflow conditions. This technique was proposed by the same authors in an...
earlier study on numerical simulation of plane channel flow (see Barri et al., 2009) and consists of recycling finite-length time series \( \tau_i \) of instantaneous velocity planes at the input. These profiles were taken from a precursor simulation and a physical constraint was introduced on \( \tau_i \) to be of order of the large-eddy-turnover time. The authors kept the inflow time series discontinuous and showed that this discontinuity in the inflow signal vanishes due to the nonlinear interactions of the turbulent flow.

Due to the principle of mass conservation, the Reynolds number in pressure-driven plane channel flow with a sudden one-sided expansion remains the same downstream of the step as in the upstream part of the channel. In a BFS Couette flow, on the other hand, the Reynolds number becomes higher downstream of the step. It is well known that the shear-driven turbulent Couette flows (see e.g. Bech et al., 1995) exhibit a number of characteristic features which make them distinguishingly different from the pressure-driven Poiseuille flow, notably the monotonically increasing mean velocity profile. The only investigation of BFS Couette flow we are aware of is the recent experimental study by Morinishi (2007). He considered a configuration with the step height \( h \) equal to half of the upstream channel height, i.e. with an expansion ratio 3:2. The Reynolds number based on the wall-friction velocity at the input was fixed to 300. This configuration (i.e. fixed \( ER \) and \( Re_i \)) allowed the author to investigate the effect of the non-dimensional upstream pressure gradient \( \beta_u \). By varying \( \beta_u \), Morinishi (2007) set up conditions for the following fully-developed upstream flows: the pure Poiseuille flow, the mixed Couette–Poiseuille flow and the pure Couette flow. For the three cases, the reattachment lengths were almost identical and about 6.5h and 2h for the primary and secondary recirculation regions, respectively.

In the present study we perform direct numerical simulation (DNS) of turbulent Couette flow over a BFS. This will enable us to gather accurate mean flow and turbulence statistics throughout the flow domain, as well as to explore in detail the instantaneous vortex topology in the shear-layer and the recirculation bubble as well as in the re-development zone. We intentionally considered a BFS configuration, where the flow upstream of the step is the same as that studied by Bech et al. (1995), i.e. a fully-developed turbulent Couette flow.

2. Flow configuration and governing equations

Fig. 1 shows a schematic view of the Couette backward-facing step flow which is composed of a step of height \( h \) and an upper-wall moving with velocity \( U_w \). Of particular relevance in backward-facing step flows is the expansion ratio \( ER \). This dimensionless parameter is defined as the ratio between the downstream and upstream channel heights, i.e. \( ER = H/(H-h) \). In the present study we consider a flow configuration, where the step height is equal to that of the upstream channel, i.e. \( H = 2h \). This gives an expansion ratio of 2:1.

The governing equations are the time-dependent, incompressible Navier–Stokes equations for a viscous fluid expressed in non-dimensional form:

\[
\nabla \cdot \mathbf{u} = 0.
\]

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}.
\]

Here, the variables have been non-dimensionalized by \( h \) and \( U_w \) and the Reynolds number based on the step height and upper-wall velocity, \( Re = U_w h/v \), is 5200.

3. Numerical approach

The computational domain has a length of \( L_x = 39h \) in the streamwise \( x \)-direction including an inlet section \( L_i = 15h \), \( H = 2h \) in the wall-normal \( y \)-direction, and \( L_z = 9.43h \) in the spanwise \( z \)-direction. A total of \( (672 \times 384 \times 192) \) grid points are used in \( x \), \( y \) and \( z \), respectively. In order to adequately resolve the turbulence scales in the separation region and the vicinity of the walls, a non-uniform mesh distribution is used in the streamwise and wall-normal directions. Thus for a viscous length scale of \( l_v = \sqrt{\nu/\zeta} \), based on the wall-friction velocity at the input \( U_i = 0.032U_w \), the first grid point next to the walls is at \( y^+ = 0.083 \) while the largest grid spacing is about \( \Delta y^+ \approx 2.4 \) (measured in wall units). For the streamwise direction, the minimum grid spacing \( \Delta x^+ \approx 4.8 \) is at the step corner and maximum at the beginning and the end of the domain with \( \Delta x^+ \approx 14.8 \). A uniform mesh is used in the spanwise \( z \)-direction with \( \Delta z^+ \approx 8.2 \). The grid specification in the inlet section is given in Table 1 together with that of Bech et al. (1995).

No-slip boundary conditions are imposed on the solid surfaces in the domain. The flow in the spanwise direction is assumed to be statistically homogeneous and periodic boundary conditions are imposed. A realistic fully turbulent flow is generated at the input by recycling finite-length time series of the instantaneous velocity planes. This technique was first used by Barri et al. (2009) in a numerical simulation of plane channel flow. The length of the time series for the current simulation was \( 33h/U_w \). This is consistent with the recommendation of Barri et al. (2009). For a plane Poiseuille flow, they demonstrated that the potential periodicity introduced by the recycling of a finite-length time series vanished when the duration of the time series equaled the large-eddy-turnover-time \( h/\tau_u \). The imposition of dynamic Dirichlet conditions at the inflow implies that the flow rate is fixed to that of the precursor simulation. The latter was that of plane Couette flow at exactly the same Reynolds number as in the extensive investigation by Bech et al. (1995). As an outflow condition, we solve the convective equation \( \partial \mathbf{u}/\partial t + U_w \partial \mathbf{u}/\partial x = 0 \) at the exit plane and set the total normal stress acting on it to zero by means of a pressure boundary condition \(-p + 2\mu \partial \mathbf{u}/\partial x = 0 \). The convective boundary condition was used in previous numerical simulations by Lowery and Reynolds (1986) for a mixing-layer and Le et al. (1997) and Barri et al. (2010) for turbulent flow over a backward-facing step and is considered suitable for vortical structures moving out of the domain.

The DNS code used to numerically solve the governing Eqs. (1) and (2) is MGLET (see Manhart, 2004). MGLET is a finite-volume code in which the Navier–Stokes equations are discretized on a
staggered Cartesian mesh with non-equidistant grid spacing. The discretization is second-order accurate in space. A second-order explicit Adams–Bashforth scheme is used for the time integration. The Poisson equation for the pressure is solved using a multi-grid algorithm.

The simulations were started from an arbitrary flow field and thereafter let to evolve to a statistically steady state. The time step used was \( \Delta t = 0.001 h/U_w \). Statistics were gathered for \( 396 h/U_w \) after the flow field first had evolved into a statistically steady state.

Direct numerical simulation of turbulent plane Couette flow has been performed in several studies. Before the statistical results of the simulation in the downstream part of the channel are presented, it is interesting to compare some primary statistics obtained in the inlet section with the fully-developed Couette flow data from Bech et al. (1995) for the same Reynolds number. This is to ensure that the finite-length time series of velocities recycled at the input provided realistic fully-developed turbulence in the upstream section.

In Fig. 2a, the mean velocity profile is plotted at \( x/h = 7 \) and is nearly indistinguishable from that of Bech et al. (1995). The computed turbulent intensities (the root-mean-square of the velocity fluctuations) are shown in Fig. 2b. The agreement between the two simulations is very good for the wall-normal and streamwise components while the streamwise component exhibits a somewhat higher turbulence level in the present case. This modest difference can be due to different domain sizes and to different grid spacings. The recent DNS of plane Couette flow at the same Reynolds number by Holstad et al. (2010) was performed with a finer mesh in a longer and wider domain. The present results in Fig. 2 are almost indistinguishable from their data.

The Kolmogorov length scale \( \eta \) was estimated from the energy dissipation rate \( \varepsilon \) as \( (\varepsilon/\nu)^{1/4} \). The adopted grid spacing turned out to never exceed \( 2\eta \).

### 4. Flow structures

In this section, we focus on the instantaneous features of turbulent Couette flow over a backward-facing step. The various plots of the velocity and vorticity fields presented in this section have been non-dimensionalized by \( U_w \) and \( U_w/h \), respectively.

#### 4.1. Velocity

A snapshot of the flow field is shown in Fig. 3, where a three-dimensional side view of the iso-surfaces of the streamwise velocity is plotted in Fig. 3a, and a plane parallel to the bottom wall at \( y' = 0.083 \) is shown in Fig. 3b. An overall picture of the separation scenario can be deduced from the figure. Starting from the inlet, the incoming flow separates at the sharp step edge and reattaches further downstream leading to the formation of a primary recirculation region. Patches of positive fluid velocity (\( u < 0.01 \)) are observed in the region with backward motion (\( u < 0 \)) and adjacent to the step in Fig. 3a, thereby identifying a secondary recirculation region. The interface between the two streams with different velocities leads to a complex three-dimensional flow structure. Fig. 3b provides a qualitative impression of the three-dimensional flow pattern close to the lower-wall. Around \( x/h \approx 22 \), it is readily observed that the reattachment location is not confined to a fixed streamwise position. Irregular bursts of fluid are formed by the shear-layer at the moment it strikes the bottom plane. Downstream the reattachment, the flow sustains its irregular pattern as it is convected towards the recovery region. Meanwhile, the primary recirculation region shows large structures that are aligned in the spanwise direction with spots of positive fluid velocity observed in-between. These tiny lumps with \( u > 0 \) are quenched by the surrounding backflow which is driven by the adverse pressure gradient.

A cross-sectional view of the individual velocity fluctuations is shown in Fig. 4. These quantities are defined as the difference between the instantaneous velocity field \( \bar{u}_i(x, y, z, t) \) and its time-spanwise average \( \bar{U}_i(x, y) \). The results show that fluctuations reach around 20% of the wall velocity \( U_w \) in the shear-layer and the primary recirculation zone indicating strong spanwise motions in these regions. The alternating positive—negative pattern observed for \( u \) downstream of the step indicates the presence of spanwise vortices. In Fig. 4c, the pseudocolors of \( w \) show large elongated structures that are inclined with respect to the streamwise direction and persist into the recovery region almost filling the entire wall gap.

In Fig. 5, we display a top view of iso-surfaces of positive and negative streamwise and spanwise fluctuations. Note that the iso-surfaces of \( w \) correspond to the instantaneous spanwise velocity since there is no mean flow in the \( z \)-direction. The flow in Fig. 5a clearly features streamwise streaks which are generated in the vicinity of the stepped wall upstream of the corner. These streaky
structures grow in width downstream of the step and remain discernible even 15h downstream of the corner. In the same region, the iso-surfaces of w show lumps of positive and negative motions. These fluctuations do not exhibit the same streamwise coherence as the streamwise velocity fluctuations.

4.2. Vorticity

Transverse vortices are a commonly observed feature in backward-facing step flows, where the underlying mechanism in the formation of such vortical structures downstream of the step is a Kelvin–Helmholtz instability. These instabilities arise from the interaction between the shear-layer and the recirculating region near the step. This interface between the high- and low-speed fluid region, being unstable, leads to such a vortex formation. In order to see whether or not K–H vortices are embedded in the present flow field, the instantaneous iso-surfaces of enstrophy are plotted in an (x,y)-plane in Fig. 6. There is an apparent roll-up of the shear-layer behind the step edge, where the unsteady K–H vortices are generated and break up into numerous small high-intensity vortices as they are transported downstream.

All turbulent flows exhibit high levels of vorticity fluctuations. The instantaneous vorticity \( \omega \) is decomposed into a mean vorticity and vorticity fluctuations:

\[
\omega = \bar{\omega} + \omega', \quad \bar{\omega} = 0.
\]

The present mean flow is statistically two-dimensional and entirely in the (x,y)-plane. Then, \( \bar{\Omega}_x \) and \( \bar{\Omega}_y \) are zero, so that the only non-zero mean vorticity component is \( \bar{\Omega}_z \). Pseudocolors of vorticity fluctuations are shown in Fig. 7. In contrast with the rather different patterns of the streamwise, wall-normal and spanwise velocity components in Fig. 4, the topology of the three components of the fluctuating vorticity vector in Fig. 7 is rather similar. Since the fluctuating vorticity field is associated with small-scale eddies, the vorticity field is only modestly affected by the anisotropic geometric constraints which enforce a substantial anisotropy on the velocity fluctuations.

A cross-stream view of the vortical structures from \( x/h = 15 \) is shown in Fig. 8a. These structures are extracted by using the \( \lambda_2 \) definition proposed by Jeong and Hussian (1995) in which a vortex core is identified as a connected region of negative \( \lambda_2 \). Here, \( \lambda_2 \) corresponds to the second largest eigenvalue of the tensor \( S_{ij}S_{ij} + \Omega_i\Omega_j \), where \( S_{ij} \) and \( \Omega_i \) represent the symmetric and antisymmetric parts of the velocity gradient tensor, respectively. In the secondary recirculation region large elongated structures are observed in contrast to the upper part of the channel that is filled with numerous small high-intensity vortices. To obtain a more detailed view on the flow in this region, the instantaneous flow in a vertical plane is shown in Fig. 8b at a distance h from the step. The highest turbulent activity is in the shear-layer which is composed of streamwise vortices, whereas in the secondary recirculation region the flow shows surprisingly strong spanwise motions.

\[
\Omega_i \quad \text{and vorticity fluctuations} \quad \omega_i \quad \text{such that} \quad \bar{\omega}_i = \Omega_i + \omega_i, \quad \bar{\omega}_i = 0.
\]

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Although the fluid motion in the lee of the step is relatively calm, the flow is by no means laminar.

5. Mean flow and turbulence statistics

The BFS simulation was performed at $Re = 5200$ which led to $Re_s/C_25 = 166$ based on the step height $h$ and the upstream friction velocity $u_s$. Some comparisons will be made with the hot-wire data of Morinishi (2007) at $Re_s/C_25 = 300$ and expansion ratio 3:2.

5.1. Mean statistics

From pressure-driven BFS flows it is known that the unsteady behaviour of the shear-layer causes the reattachment line to fluctuate around a mean value $X_{R}$. The streamline pattern of the mean flow in Fig. 9 shows a large primary separation bubble which extends about 7.5 $h$ downstream of the step. A secondary bubble of length 1.78 $h$ can be observed adjacent to the corner. The skin friction coefficient, defined as $C_f = \tau_w/\frac{1}{2} \rho u_s^2$, is shown in Fig. 10 and confirms that a secondary separation bubble with anti-clockwise flow ($C_f > 0$) is embedded within the primary separation bubble with clockwise motion ($C_f < 0$). This flow pattern is consistent with the findings of Morinishi (2007) who reported that reattachment occurred at $X_{R} = 6.63h \pm 1.4h$ and the secondary bubble was at $1.88h \pm 0.4h$. Downstream of $x/h = 30$, $C_f$ is almost constant along both walls, but with the wall-friction along the moving surface being about 10 times higher than at the lower surface. This suggests a substantial asymmetry of the mean velocity field.

The local wall pressure coefficient is defined as

$$C_p = \frac{P - P_o}{\frac{1}{2} \rho u_s^2}$$

where $P_o$ is a reference pressure taken at $x/h = 0$. In Fig. 11, $C_p$ is shown together with the corresponding values from Morinishi (2007) for Couette flow over a backward-facing step. The agreement between the computational and experimental results is very good. The local wall pressure coefficient exhibits a local minimum close to the position of maximum backflow (i.e. beneath the core of the primary separation bubble), just as in the pressure-driven BFS flow; Barri et al. (2010). Downstream of $x/h = 30$, an almost linear variation of $C_p$ is observed for the two cases. This implies that the streamwise mean pressure gradient has become independent of $x$, in keeping with the measurements of Morinishi (2007), and the flow field can be considered as being nearly fully-developed in the downstream part of the computational domain. This is also consistent with the constancy of $C_f$ observed in Fig. 10.

The mean velocity profiles are presented in Figs. 12 and 13 for the streamwise and wall-normal components at different representative locations: inside the secondary bubble, through the primary recirculation, downstream of the reattachment and in the recovery region. In the recirculation region the strongest backflow is observed beneath the core of the primary bubble, whereas the secondary separation region shows a weak mean-streamwise...
motion. Although the characteristic S-shape of the mean velocity profile $U(x,y)$ has been retained at $x/h = 27$, the profile is yet far from being anti-symmetric. Midway between the walls $U$ is still roughly half of $U_w/2$ which should have been reached in the case a fully redeveloped Couette flow. However, irrespective of the length of the domain that can be used in the downstream part of the channel, an anti-symmetric profile corresponding to a fully redeveloped Couette flow will not be reached. This is due to the principle of mass conservation. It follows that since the height of the domain after the step is twice that of the inlet section and the mean velocity profile of Couette flow is monotonically increasing to a constant value of $U_w$, the flow cannot adjust itself to an anti-symmetric S-profile shape and at the same time maintain a constant flow rate.

An examination of the mean wall-normal velocity profiles $V(x,y)$ presented in Fig. 13 shows that the flow emanating from the step corner exhibits an upward motion in the lower half of the channel and a downward flow above $y/h \approx 1$. The distinct positive $V$ at $x/h = 16$ results from the primary recirculation bubble, in which the return flow has separated from the wall at $x/h \approx 16.8$. This pattern is sustained up till $x/h \approx 18$, where afterwards a relatively strong downward motion is solely observed above the remaining part of the recirculation zone. Downstream of the reattachment region, the mean wall-normal velocity is attenuated.

5.2. Turbulence intensities and Reynolds shear stress

The turbulence intensities and the Reynolds shear stress are shown in Figs. 14 and 15 at different streamwise locations downstream the step. The $rms$ values and $-\overline{uv}$ exhibit a high turbulence level immediately downstream of the corner of the step at $y/h \approx 1$. This localized high turbulence zone, mainly for $urms$, is obviously caused by the locally high mean-shear-rate in the shear-layer emanating from the step edge. As the flow progresses downstream, the streamwise turbulence intensity peaks are broadened and attenuated while the turbulence levels of the spanwise and wall-normal components increase. Downstream the reattachment and in the recovery region, the asymmetry in $urms$ persists, where a substantially higher longitudinal turbulence intensity is observed near the moving wall that is almost twice that seen near the stationary wall. Apart from the secondary recirculation region, where $wrms > urms > vrms$, the turbulence exhibits everywhere the usual shear-flow anisotropy with the streamwise intensity being the most significant. The flow anisotropy was not accessible with the L-type hot-wire used in Morinishi’s (2007) measurements since only $urms$ could be obtained.

The profiles of $-\overline{uv}$ show that the Reynolds shear stress is positive almost throughout the whole domain. The distinct positive peak of $-\overline{uv}$ is typical of a turbulent mixing-layer. The peak broadens with downstream distance but remains discernible above the

![Fig. 12. Mean streamwise velocity profiles. (a) Recirculation region and (b) recirculation, reattachment and recovery regions.](image-url)

![Fig. 13. Mean wall-normal velocity profiles. (a) Recirculation region and (b) recirculation, reattachment and recovery regions.](image-url)

![Fig. 14. Turbulent intensities scaled with the maximum mean velocity at the input: — — — —, streamwise direction; — — — —, wall-normal direction; — — — —, spanwise direction. (a) Recirculation region and (b) reattachment and recovery regions.](image-url)

![Fig. 15. Reynolds shear stress profiles. (a) Recirculation region and (b) reattachment and recovery regions.](image-url)
Reynolds shear stress $-\overline{u'v'}$. (a) Recirculation region and (b) reattachment and recovery regions.

Reattachment point at about 7h downstream of the step. Further downstream, the $-\overline{u'v'}$ profile fails to attain the symmetric shape which characterizes a pure Couette flow; see e.g. Bech and Andersson (1996). The profile at $x/h = 37$, i.e. 22 step heights downstream of the step, shows that the Reynolds shear stress is substantially higher near the moving wall than along the fixed wall.

5.3. Reynolds stress budget

In this section, the Reynolds stress budget is presented at a distance $h$ from the step and in a wall-normal region confined to the shear-layer region (i.e. $0.5 < y/h < 1.5$). The transport equation for the Reynolds stress tensor is:

$$\frac{D}{Dt} \left( \overline{u_j u_i} \right) = P_{ij} - e_{ij} + \Pi_{ij} + G_{ij} + D_{ij} + T_{ij}$$

where the production, dissipation, pressure–strain, pressure diffusion, molecular diffusion and turbulent diffusion are defined as:

$$P_{ij} = -\overline{u_i \partial u_j / \partial x_k} - \overline{u_j \partial u_i / \partial x_k}$$

$$e_{ij} = 2\overline{\partial u_i \partial u_j / \partial x_k \partial x_k}$$

$$\Pi_{ij} = \frac{1}{\rho} \left( \partial \overline{u_i \partial u_j / \partial x_k} + \partial \overline{u_j \partial u_i / \partial x_k} \right)$$

$$G_{ij} = -\frac{1}{\rho} \left( \partial \overline{u_i \partial u_j / \partial x_k} + \partial \overline{u_j \partial u_i / \partial x_k} \right)$$

$$D_{ij} = \nu \frac{\partial^2 \overline{u_i u_j}}{\partial x_k \partial x_k}$$

$$T_{ij} = -\frac{\partial}{\partial x_k} \overline{\rho \overline{u_i u_j} U_k}$$

The budget for $\overline{u'v'}$ in Fig. 16a is largely dominated by production and pressure–strain, where a large peak of negative production is observed close to the position of maximum $u_{max}$ (i.e. $y/h \approx 1$). Meanwhile, the profiles of the molecular diffusion in Fig. 16 show that the contribution of $D_{ij}$ is negligible everywhere and thereby turbulent and pressure diffusion play the role of transporting energy across the shear-layer. For the normal stresses, $T_{ij}$ carries energy away from the position of maximum production.

The mean-shear production $P_{12}$ in Fig. 16b is clearly smaller than $P_{11}$ in Fig. 16a, but nevertheless far from being negligible as in free shear flows. In the present case $P_{12}$ stems from $-\overline{u'v'} / \partial y$, which attains non-vanishing levels due to the underlying recirculating flow; see the V-profile in Fig. 13a. Since there is only modest production of $\overline{u'v'}$, the primary source of energy is from $\Pi_{ij}$ which serves to redistribute energy between the normal stresses. Across the entire shear-layer, $\overline{u'v'}$ and $\overline{w'w'}$ act as receiving components taking energy from $\overline{u'v'}$. Moreover, the maximum rate of energy transfer is reported at the peak production for the streamwise normal stress.

As in the budget of $\overline{u'v'}$, the equation for $\overline{u'w'}$ is largely dominated by production and pressure–strain, where a large peak of negative production is observed around $y/h = 1$. The expression for this production term is

$$P_{12} = -\overline{u'v' \partial u_j / \partial x_k} - \overline{u_j \partial (u'v') / \partial x_k} - \overline{w' \partial (u'v') / \partial x_k} - \overline{\rho \partial v' / \partial x_k}$$

where the mean flow is statistically two-dimensional, the first and fourth terms of Eq. (6) add up to zero due to mass conservation. This implies that the sign of $P_{12}$ depends solely on the gradients of the mean streamwise and wall-normal velocities. Since $\partial u_j / \partial y$ being dominant over all the other mean gradient terms in this region, then Eq. (6) reduces to $P_{12} \sim -\overline{u'v' \partial u_j / \partial x_k}$. The production of the Reynolds shear stress is thus governed by $\partial u_j / \partial y$ and peak production is attained, where $\partial u_j / \partial y$ exhibits a local maximum in the shear-layer. This is consistent with a free mixing-layer.

The turbulent diffusion $T_{12}$ contributes to the increase of $\overline{u'v'}$ in the central region whereas pressure diffusion $G_{12}$ transports $\overline{u'v'}$ downwards from the upper part of the shear-layer. Meanwhile, viscous dissipation is negligible almost everywhere in this case. The nearly negligible viscous dissipation is consistent with the observation made by Bech and Andersson (1996) in a fully-developed Couette flow. This is because $e_{12}$ consists of relatively weakly correlated velocity gradients $\partial u_j / \partial x_k$ and $\partial u_k / \partial x_j$. This is in contrast to the diagonal components of $e_{ij}$ which do not involve cross-correlations.

Although the convection term $\partial_k (\overline{u_i u_j}) / \partial x_k$ is almost negligible for the stress components dominated by production (i.e. $\overline{u'v'}$ and $\overline{u'w'}$), it shows significant positive values in the shear-layer for the wall-normal and spanwise components. In fact, the convective transport of $\overline{u'w'}$ is equally important as its dissipation rate $e_{33}$.

5.4. Recovery region

The two-dimensional mean flow has developed to an essentially unidirectional flow in the downstream part of the computational domain, i.e. beyond $x/h = 30$ or 15 step heights downstream of the sudden expansion. It is noteworthy that the upstream pure Couette flow redeveloped into a mixed Couette–Poiseuille flow in contrast to the classical pressure-driven backward-facing step flow, where an upstream Poiseuille flow inevitably redevelops to another pure Poiseuille flow far downstream of the step. In the present case, however, an adverse pressure gradient is established with the view to assure global mass conservation. The resulting mixed Couette–Poiseuille flow exhibits major asymmetries in the turbulence field with a substantially reduced turbulence level along the stationary wall, and the flow field closely resembles the Couette–Poiseuille flow simulations reported by Kuroda et al. (1995).

Kuroda et al. (1995) conducted direct numerical simulations of fully-developed turbulent plane Couette–Poiseuille flow to study the effect of the mean-shear-rate on the near-wall turbulent flow.
field. Four cases were considered by these authors in which the mean-shear-rate was varied by changing the wall velocity and the streamwise pressure gradient. Of particular relevance to our study is case CP3 in Kuroda et al. (1995).

Owing to the asymmetry of the Couette–Poiseuille flow, the wall shear stresses on the lower and upper-wall are generally different. For the current simulation, the corresponding local Reynolds numbers based on the local wall friction-velocities and the non-dimensionalized pressure gradient are provided in Table 2 together with simulation parameters from Kuroda et al. (1995). The mean velocity profile and turbulent intensities presented in Fig. 17 compare surprisingly well with the DNS data of Kuroda et al. (1995). The rather different behaviour in the vicinity of the lower-wall is caused by the DNS data of Kuroda et al. (1995), especially near the upper-wall. However, substantial differences are observed near the lower-wall. The streamwise turbulence intensity, for instance, exhibits a modest peak in the present case, whereas a monotonic increase towards the peak near the upper-wall is exhibited by the DNS data of Kuroda et al. (1995). The friction velocity is substantially reduced by the adverse pressure gradient but yet of sufficient magnitude to give rise to mean-shear production of streamwise turbulence fluctuations. The Reynolds stress budget presented by Kuroda et al. (1995) indeed showed that mean-shear production only plays a marginal role near the lower-wall.

5.5. Vorticity and $\lambda_2$ statistics

Profiles of the root-mean-square vorticity normalized by $u_0^2/v$ are shown in Fig. 18. While the secondary recirculation region in Fig. 18a shows low levels, the effect of mean-shear motion is clearly observed above $y/h \approx 0.75$ in the form of increasing vorticity fluctuations. This increase is sustained till midway between the walls, where the three components attain their local maxima in the shear-layer, followed by a decrease towards a local minimum around $y/h \approx 1.5$. Two step heights further downstream, the primary recirculation region in Fig. 18b shows higher fluctuation levels than the secondary one while in the shear-layer $(\omega_x^2)^{1/2}$ is amplified and $(\omega_y^2)^{1/2}$ is attenuated. In a direct numerical simulation of a self-similar mixing-layer Rogers and Moser (1993) reported an axisymmetric anisotropy in the free planar shear-layer. In the present case, however, the results show that the vorticity is almost isotropic in the shear-layer. Near the upper-wall however, the distributions of $(\omega_y^2)^{1/2}$ are highly anisotropic as in the case of plane Couette flow, whereas at the stationary wall a different anisotropy is observed with $(\omega_x^2)^{1/2}$ and $(\omega_z^2)^{1/2}$ being almost equal and attaining higher values than $(\omega_y^2)^{1/2}$. It is noteworthy that the flow exhibits the same anisotropy further downstream. The wall-normal vorticity fluctuations inevitably vanish at both walls due to the no-slip conditions. The local near-wall peak of $(\omega_y^2)^{1/2}$, which is associated with streamwise-oriented vortices, is absent along the fixed wall.

Fig. 19 shows the cross-correlation between $-\lambda_2$ and the three components of the vorticity fluctuation vector in the recirculation and recovery regions. The main tendency at both $x$-stations is that the cross-correlation between $-\lambda_2$ and $\omega_z$ is higher than that between $-\lambda_2$ and $\omega_x$ or $\omega_y$. This indicates that the turbulent vortices are predominantly aligned in the streamwise direction, even though the fluctuating vorticity field, associated with the small-scale turbulence, is close to isotropy.

![Fig. 16. Reynolds stress budget normalized by $u_0^2/v$ at $x/h = 16$.](image)

![Table 2](image)

<table>
<thead>
<tr>
<th>Case</th>
<th>$U_0H/2v$</th>
<th>$c \times \partial P/\partial x$</th>
<th>$w_H/2v$</th>
<th>$w_0H/2v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kuroda</td>
<td>3000</td>
<td>$-1.33 \times 10^{-3}$</td>
<td>154</td>
<td>17.7</td>
</tr>
<tr>
<td>Present</td>
<td>5200</td>
<td>$1.2 \times 10^{-3}$</td>
<td>233.8</td>
<td>77</td>
</tr>
</tbody>
</table>
Fig. 17. (a) Mean velocity profile $U/U_w$ at $x/h = 37$. (b) Turbulent intensities at $x/h = 37$: ---, $\omega$, streamwise direction; --., $\omega$, wall-normal direction; ----, $\phi$, spanwise direction. The symbols denote DNS data from Kuroda et al. (1995).

Fig. 18. Wall-normal distribution of the root-mean-square vorticity fluctuations normalized by $u_s^2$: — — — —, streamwise direction; ••••, wall-normal direction; — — — , spanwise direction. (a) $x/h = 16$; (b) $x/h = 18$; (c) $x/h = 22$; (d) $x/h = 37$.

Fig. 19. Wall-normal distribution of the dimensionless cross-correlation of $-\omega_2$ with $\omega_3$ normalized by $u_s^2/\nu$: ---, $-\omega_2\omega_3$; ••••, $-\omega_2\omega_3$; ----, $-\omega_2\omega_3$. (a) $x/h = 16$ and (b) $x/h = 37$. 
6. Conclusions

A direct numerical simulation of turbulent Couette flow over a backward-facing step has been performed at a relatively low Reynolds number. The mean reattachment length of the shear-layer was found to be 7.5h. In the recirculation zone a large negative skin friction coefficient was observed beneath the core of the primary separation bubble.

The streaky near-wall structures formed along the step persisted several step heights downstream of the step. High and anisotropic turbulence levels were produced in the shear-layer emanating from the corner. Although advection by the mean flow as well as turbulent diffusion were of some importance in the vicinity of the step, the mean-shear production was the primary source of turbulent energy. As in other turbulent shear flows, the energy arose as streamwise velocity fluctuations and was subsequently transferred into wall-normal and spanwise fluctuations by means of pressure–strain interactions.

The shear-layer which formed when the upstream Couette flow mixed with the recirculating flow downstream of the step resembles, at least qualitatively, a free mixing-layer in many respects. However, the turbulence in the approaching Couette flow is convected into the early stages of the mixing zone, where also the “upwelling” caused by the primary separation bubble has a major influence. Beyond 3h downstream of the corner, the mean flow turns towards the reattachment point and the shear-layer is simultaneously affected by the streamwise curvature and an adverse mean pressure gradient. The shear-layer which develops from the corner of the BFS is accordingly exposed to a number of different influences not present in the freely developing mixing-layer.

Even though the wall-normal mean velocity V vanished far downstream of the step, the streamwise velocity U did not retain the characteristic S-shape typical of a pure Couette flow. This phenomenon is ascribed to the principle of global mass conservation, which can be fulfilled only if an adverse mean pressure gradient is established in the recovery region. This is indeed what was observed. In the resulting asymmetric mean flow, the wall-friction was roughly 10 times higher along the moving wall than at the steady surface. The resulting flow field was thus by far less vigorous along the stepped wall and this was reflected in the low levels of velocity and vorticity fluctuations.

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References


