Numerical and experimental investigations into the application of response conditioned waves for long-term nonlinear analyses

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ABSTRACT

The coefficient of contribution method, in which the extreme response is determined by considering only the few most important sea states, is an efficient way to do nonlinear long-term load analyses. To furthermore efficiently find the nonlinear short-term probability distributions of the vessel responses in these sea states, response conditioned wave methods can be used. Several researchers have studied the accuracy of response conditioned wave methods for this purpose. However, further investigations are necessary before these can become established tools. In this paper we investigate the accuracy by comparing the short-term probability distributions obtained from random irregular waves with those from response conditioned waves. We furthermore show how response conditioned wave methods can be fitted into a long-term response analysis. The numerical and experimental investigations were performed using a container vessel with a length between perpendiculars of 281 m. Numerical simulations were done with a nonlinear hydroelastic time domain code. Experiments were carried out with a flexible model of the vessel in the towing tank at the Marine Technology Centre in Trondheim. The focus was on the probability distributions of the midship vertical hogging bending moments in the sea states contributing most to the hogging moments with a mean return period of 20 years and 10 000 years. We found that the response conditioned wave methods can be...
1. Introduction

According to Airy wave theory, the most unfavourable wave condition for a vessel is not the wave of which all components have a peak at the same time instant, but the wave leading to a response for which this is the case. Using the ideas from Friis-Hansen and Nielsen [1], Taylor et al. [2] and Adegeest et al. [3], Dietz [4] formulated the most likely response wave (MLRW). Using the amplitude and phase angle information from the transfer function, this method uses linear theory to condition the incident wave profile to cause a predefined specific response at a certain time instant. At this time all of the responses' frequency components have a peak. The most likely response wave is a special case of the conditional random response wave (CRRW), which was reformulated by Dietz [4], based on the original idea of Taylor et al. [2]. The CRRW profile includes a random background wave within the MLRW profile, and is therefore more capable of correctly capturing the contribution of transient effects like those of whipping.

Using the conditioned wave sequence as input, time domain simulations or model tests can be carried out. Pastoor [5] showed that response conditioned wave methods can be used for predicting nonlinear short-term probability distributions of the midship vertical bending moments in an efficient manner. By combining these methods with the coefficient of contribution method described for instance by Baarholm and Moan [6] a very efficient way is found to predict the long-term extreme response. The coefficient of contribution method identifies the most important sea states contributing to the extreme response, and Baarholm and Moan [6] demonstrated that a nonlinear long-term load analysis for marine structures can be performed by considering only these few most important sea states. The accuracy with which the short-term probability distributions in these sea states can be predicted by response conditioned wave methods is then of interest.

Assuming a rigid hull girder, Dietz [4] compared nonlinear short-term probability distributions of the midship bending moments obtained using both the MLRW and the CRRW methods with results from brute force simulations, and found good agreement. He furthermore showed that the effect of the random background wave was of increasing importance for increasing vessel speed. For a flexible hull girder, bending moments obtained by the CRRW method also corresponded well with brute force results. The same can be said for the sagging moments by application of the MLRW method. The hogging moments, on the other hand, compared less well with results from brute force simulations. Dietz [4] stressed that he used only one time domain code and recommended further analysis with other types of vessels and comparison with results from model tests. Also, ISSC [7] recommended that response conditioning methods have to be verified with further numerical simulations and experimental results before they can become established tools.

The fundamental assumption of wave conditioning techniques is that the nonlinear response is a correction of the linear response. It is therefore important to investigate their applicability under severe conditions, when nonlinear effects are significant. The purpose of this paper is to compare the short-term probability distributions obtained from random irregular waves with those from response conditioned waves. Both the MLRW and the CRRW methods were investigated. We furthermore show how these methods can be fitted into a long-term response analysis. The numerical and experimental studies were performed using a container vessel with a length between perpendiculars of 281 m. The nonlinear hydroelastic hybrid time domain computer code developed by Wu and Moan [8] was used for the calculations. The focus was on the probability distributions of the midship vertical hogging bending moments in the sea states contributing most to the hogging moments with a mean return period of 20 years and 10 000 years. The experimental results were obtained by testing a four segment flexible model of the vessel in the towing tank at the Marine Technology Centre in Trondheim. In the
2. Theoretical background

2.1. Response conditioned waves

According to linear theory the wave elevation may be seen as the superposition of sinusoidal wave components. Seen from a coordinate system moving with the forward speed of the ship, \( U \), the wave elevation process, \( \zeta(t) \), at the centre of gravity of the ship may be written as

\[
\zeta(t) = \sum_{n=1}^{N} a_{\zeta,n}^{e} \left[ V_n \cos(-\omega_{e,n}t) + W_n \sin(-\omega_{e,n}t) \right],
\]

(1)

where \( N \) is the number of wave components, \( V_n \) and \( W_n \) are independent standard normal random variables, \( \omega_{e,n} \) is the encounter frequency, \( \omega_e \), of the \( n \)th component, and \( t \) represents time. The coefficients \( a_{\zeta,n}^{e} \) are found from the encountered wave spectrum, \( S_{\zeta}(\omega) \),

\[
a_{\zeta,n}^{e} = \sqrt{S_{\zeta}^{e}(\omega_{e,n})} \Delta \omega_{e,n},
\]

(2)

where \( \Delta \omega_{e,n} \) is the resolution of the discrete encounter frequency. The encountered wave spectrum is found from the wave spectrum, \( S_{\zeta}(\omega) \), as follows

\[
S_{\zeta}^{e}(\omega_{e}) = S_{\zeta}(\omega) \frac{d\omega}{d\omega_{e}}
\]

(3)

In head seas, the encounter frequency is related to the wave frequency, \( \omega_e \), by

\[
\omega_e = \omega + \frac{2U}{g},
\]

(4)

where \( g \) is the acceleration of gravity. For linear responses the spectrum, \( S_{M}^{e}(\omega_{e}) \), can be obtained from

\[
S_{M}^{e}(\omega_{e}) = \left[ \Phi_{M}^{e}(\omega_{e}) \right]^{2} S_{\zeta}^{e}(\omega_{e}).
\]

(5)

Here \( \Phi_{M}^{e}(\omega_{e}) \) is the response amplitude operator (RAO) of the response related to the wave at the centre of gravity of the vessel. The \( k \)th spectral moment, \( m_k \), of the response is given as

\[
m_k = \int_{0}^{\infty} \omega^k S_{M}^{e}(\omega_{e}) d\omega_{e}.
\]

(6)

The response of the ship, \( M(t) \), in the time domain is found from

\[
M(t) = \sum_{n=1}^{N} a_{M,n}^{e} \left[ V_n \cos(-\omega_{e,n}t + \theta_{M,n}^{e}) + W_n \sin(-\omega_{e,n}t + \theta_{M,n}^{e}) \right],
\]

(7)

where \( \theta_{M,n}^{e} \) denotes the phase angle of the transfer function, of the \( n \)th component. The coefficient \( a_{M,n}^{e} \) is related to \( a_{\zeta,n}^{e} \) through the RAO

\[
a_{M,n}^{e} = \Phi_{M}^{e}(\omega_{e,n}) a_{\zeta,n}^{e} = \sqrt{S_{M}^{e}(\omega_{e,n})} \Delta \omega_{e,n},
\]

(8)

where \( \Phi_{M}^{e}(\omega_{e,n}) \). Through the transfer function, \( \Phi_{M}^{e}(\omega_{e}) \) and \( \theta_{M}^{e}(\omega_{e}) \), \( M(t) \) also depends on the spatial coordinate, \( x \), of the vessel.

It is well known that narrow band processes tend to concentrate the upcrossings in clusters. The individual upcrossings can then not be assumed independent. It is therefore better to consider the envelope process, \( R(t) \), of \( M(t) \) instead of \( M(t) \) itself. Cramér and Leadbetter [9] defined the envelope process by
\[ R(t) = \sqrt{M^2(t) + \dot{M}^2(t)}, \]  
(9)

which is an envelope process of \( M(t) \) since \( R(t) \geq |M(t)| \) for all \( t \), and \( R(t) = |M(t)| \) for some \( t \) [9]. \( \dot{M}(t) \) is the Hilbert transform of \( M(t) \),

\[ \dot{M}(t) = \sum_{n=1}^{N} a_{M,n}^e \left[ - V_n \sin \left( - \omega_{e,n} t + \theta_{M,n}^e \right) + W_n \cos \left( - \omega_{e,n} t + \theta_{M,n}^e \right) \right]. \]  
(10)

The wave profile, \( \zeta(t) \), conditional on a given linear response amplitude, \( M_c \), at time \( t = 0 \) is then given by

\[ \zeta_c(t) = [\zeta(t) | M(0) = (M_c, 0), \dot{M}(0) = (0, M_c \omega_M)], \]  
(11)

with \( \omega_M \) the instantaneous response frequency, and

\[ M(t) = \begin{bmatrix} M(t) \\ \dot{M}(t) \end{bmatrix}. \]  
(12)

Considering Eqs. (7) and (10), Eq. (11) can be translated to the following four equations.

\[ \sum_{n=1}^{N} a_{M,n}^e V_{n,c} \cos \left( \theta_{M,n}^e \right) + W_{n,c} \sin \left( \theta_{M,n}^e \right) = M_c \]  
(13)

\[ \sum_{n=1}^{N} a_{M,n}^e \omega_{e,n} V_{n,c} \sin \left( \theta_{M,n}^e \right) - W_{n,c} \cos \left( \theta_{M,n}^e \right) = 0 \]  
(14)

\[ \sum_{n=1}^{N} a_{M,n}^e - V_{n,c} \sin \left( \theta_{M,n}^e \right) + W_{n,c} \cos \left( \theta_{M,n}^e \right) = 0 \]  
(15)

\[ \sum_{n=1}^{N} a_{M,n}^e \omega_{e,n} V_{n,c} \cos \left( \theta_{M,n}^e \right) + W_{n,c} \sin \left( \theta_{M,n}^e \right) = M_c \omega_M \]  
(16)

Where \( V_{n,c} \) and \( W_{n,c} \), respectively, denote the conditioned \( V_n \) and \( W_n \) for which the above equations are true. These were obtained by use of Slepian model processes, see e.g. Ditlevsen [10], Friis-Hansen and Nielsen [1], Dietz et al. [11] or Dietz [4], and are equal to

\[ V_{n,c} = V_n - \frac{a_{M,n}^e}{m_2 m_0 - m_1^2} \left[ (m_2 - \omega_{e,n} m_1) S_1 \cos \left( \theta_{M,n}^e \right) - M_c (m_2 - \omega_{e,n} m_1) \cos \left( \theta_{M,n}^e \right) \right] \\ + (\omega_{e,n} m_0 - m_1) S_2 \sin \left( \theta_{M,n}^e \right) + (\omega_{e,n} m_1 - m_2) S_3 \sin \left( \theta_{M,n}^e \right) \\ + (\omega_{e,n} m_0 - m_1) S_4 \cos \left( \theta_{M,n}^e \right) - M_c \omega_M (\omega_{e,n} m_0 - m_1) \cos \left( \theta_{M,n}^e \right) \right]; \]  
(17)

\[ W_{n,c} = W_n - \frac{a_{M,n}^e}{m_2 m_0 - m_1^2} \left[ (m_2 - \omega_{e,n} m_1) S_1 \sin \left( \theta_{M,n}^e \right) - M_c (m_2 - \omega_{e,n} m_1) \sin \left( \theta_{M,n}^e \right) \right] \\ - (\omega_{e,n} m_0 - m_1) S_2 \cos \left( \theta_{M,n}^e \right) - (\omega_{e,n} m_1 - m_2) S_3 \cos \left( \theta_{M,n}^e \right) \\ + (\omega_{e,n} m_0 - m_1) S_4 \sin \left( \theta_{M,n}^e \right) - M_c \omega_M (\omega_{e,n} m_0 - m_1) \sin \left( \theta_{M,n}^e \right) \right]. \]  
(18)

Here \( S_1, S_2, S_3 \) and \( S_4 \) are

\[ S_1 = \sum_{n=1}^{N} a_{M,n}^e V_n \cos \left( \theta_{M,n}^e \right) + W_n \sin \left( \theta_{M,n}^e \right) \]  
(19)
Details of the above derivation were presented by Dietz et al. [11] and Dietz [4]. The correctness of Eqs. (17) and (18) is easily verified by substituting them in Eqs. (13)–(16).

By substituting $V_{n,c}$ and $W_{n,c}$ in Eq. (1) the CRRW profile is obtained. The MLRW profile is found by substitution of the mean values of $V_{n,c}$ and $W_{n,c}$. $\nabla_{n,c}$ and $\mathbf{W}_{n,c}$ respectively, in Eq. (1),

$$\nabla_{n,c} = \frac{\alpha_{M,n} M_c}{m_2 m_0 - m_1^2} [(m_2 - \omega_{e,n} m_1) + \omega_M (\omega_{e,n} m_0 - m_1)] \cos \left( \theta_{M,n} \right);$$

(23)

$$\mathbf{W}_{n,c} = \frac{\alpha_{M,n} M_c}{m_2 m_0 - m_1^2} [(m_2 - \omega_{e,n} m_1) + \omega_M (\omega_{e,n} m_0 - m_1)] \sin \left( \theta_{M,n} \right).$$

(24)

From Eqs. (23) and (24) it is easily seen that under certain restrictions

$$\arctan \left( \frac{\mathbf{W}_{n,c}}{\nabla_{n,c}} \right) = \theta_{M,n}.$$  

(25)

By substituting $\nabla_{n,c}$ and $\mathbf{W}_{n,c}$ in Eq. (7) it can then be verified that all components have a peak at $t = 0$. Consequently the response is symmetric around this time instant. By a simple transformation the specified response can however be made to occur at any other time instant. Fukasawa et al. [12] used the idea that all response components have a peak at the same time instant in a more direct manner to derive the underlying design irregular wave. Their approach is similar to the MLRW method, but in their case the wave is not conditioned on a particular response.

In numerical methods the incident wave is often given in the centre of gravity of the vessel. In these cases $\zeta(t)$, as obtained above, can be used directly. For model tests on the other hand both the time of occurrence of the conditioned event and its location in the tank should be chosen. $\zeta(t)$ then has to be transformed from the target location to that of the wave maker.

If $\omega_M = m_1/m_0$ the MLRW profile is the same as the most likely extreme response (MLER) wave developed by Adegeest et al. [3]. This expression is used throughout this paper.

It should be noted that $\alpha_{M,n}$ is proportional to the significant wave height, $H_s$. The spectral moments, on the other hand, are proportional to $H_s^2$. This means that $\nabla_{n,c}$ and $\mathbf{W}_{n,c}$ are inversely proportional to $H_s$. Combined with the fact that $\alpha_{e,n}$ is again proportional to $H_s$ this means the MLRW profiles are independent of $H_s$. Furthermore, $\nabla_{n,c}$ and $\mathbf{W}_{n,c}$ are proportional to $M_c$. A change in $M_c$ will thus only cause a change in the height of the conditioned wave, not in its shape.

Finally, it should be mentioned that, in order for the CRRW profile to be correct, it is important that the numerical integration in Eq. (6) is performed such that the following equations are true.

$$m_0 = \sum_{n=1}^{N} a_{M,n}^2$$

(26)

$$m_1 = \sum_{n=1}^{N} a_{M,n}^2 \omega_{e,n}$$

(27)
\[ m_2 = \sum_{n=1}^{N} \sigma_{M,n}^2 \sigma_{e,n}^2 \]  

(28)

2.2. Long-term load analyses and the coefficient of contribution method

The long-term distribution of peaks may be obtained by summing short-term probability distributions in sea states covering all possible combinations of peak periods, \( T_p \), significant wave heights, \( H_s \), forward speeds, \( U \) and heading angles, \( \beta \). The long-term probability that a maximum, \( R \), of the response will be larger than \( r \) is then written as

\[ P(R > r) = \int_{H_i}^{H_f} \int_{T_p}^{T_f} \int_{U}^{U_f} \int_{\beta}^{\beta_f} P(R > r| h, t, u, \beta) f_{H_i, T_p, U, \beta}(h, t, u, \beta) \overline{W} \, dh \, dt \, du \, d\beta, \]

(29)

where \( \overline{W} \) is a weight function which expresses the relative rate of peak responses within each sea state. \( f_{H_i, T_p, U, \beta}(h, t, u, \beta) \) is the joint long-term probability distribution function of the significant wave heights, peak periods, forward speeds and heading angles. \( P(R > r| h, t, u, \beta) \) is the short-term cumulative probability distribution of the response in the sea state characterised by a significant wave height \( h \) and peak period \( t \), in which the vessel has a forward speed \( u \) and a heading angle \( \beta \).

When only one heading and forward speed are considered Eq. (29) simplifies to

\[ P(R > r) = \int_{H_i}^{H_f} \int_{T_p}^{T_f} P(R > r|h, t) f_{H_i, T_p}(h, t) \overline{W} \, dh \, dt. \]

(30)

which may be discretised as

\[ P(R > r) = \sum_{i} \sum_{j} P(R > r|h_i, t_j) f_{H_i, T_p}(h_i, t_j) \overline{W}_{ij} \Delta h \Delta t. \]

(31)

Here \( \overline{W}_{ij} \) is the weight function for a sea state with significant wave height \( h_i \) and peak period \( t_j \).

The initial step in the efficient long-term analysis as proposed in the Introduction is to perform a complete linear long-term analysis, and to find the response, \( r_D \), with a mean return period of \( D \) years. This can be done very efficiently in the frequency domain. Using Eq. (31) \( r_D \) is found by solving

\[ P(R > r_D) = \sum_{i} \sum_{j} P(R > r_D|h_i, t_j) f_{H_i, T_p}(h_i, t_j) \overline{W}_{ij} \Delta h \Delta t = \frac{1}{N_D}. \]

(32)

where \( N_D \) is the average number of response cycles in \( D \) years.

The coefficients of contribution, \( C_{R,ij} \), of the sea state with significant wave height \( h_i \) and peak period \( t_j \) can then be written as

\[ C_{R,ij} = \frac{P(R > r_D|h_i, t_j) f_{H_i, T_p}(h_i, t_j) \overline{W}_{ij} \Delta h \Delta t}{P(R > r_D)} = N_D P(R > r_D|h_i, t_j) f_{H_i, T_p}(h_i, t_j) \overline{W}_{ij} \Delta h \Delta t. \]

(33)

In the linear analysis, the dominating area in the scatter diagram is located in the regions where the wave length is approximately equal to the ship length. When nonlinear effects are included, the dominating area does not necessarily coincide with the area found from the linear analysis. Multiple maxima may even occur due to large responses at other periods, e.g. due to slamming, which generally occurs in slightly shorter and steeper waves. Therefore, Baarholm and Moan [6] described an iterative procedure for finding the coefficients of contribution when nonlinear effects are included. For the sea states in the dominating area of the scatter diagram found using linear theory, nonlinear time domain simulations are performed in order to find \( P(R_{NL} > r_{D;NL}|h_i, t_j) \), and a \( D \)-year long-term nonlinear response, \( r_{D;NL} \), is derived.
\[ P(R_{NL} > r_{D;NL}) = \sum_i \sum_j P(R_{NL} > r_{D;NL}|h_i, t_j)f_{H, T_p}(h_i, t_j)w_{ij} \Delta h \Delta t = \frac{1}{N_D} \]  

(34)

for all combinations of \(i\) and \(j\) for which \(C_{R;ij} > \alpha\), where \(\alpha\) is a threshold value [6]. For the long-term analysis proposed here \(P(R_{NL} > r_{D;NL}|h_i, t_j)\) is found using response conditioned waves. Dietz [4] was the first to propose combining the coefficient of contribution and response conditioned wave methods. How \(P(R_{NL} > r_{D;NL}|h_i, t_j)\) is found using response conditioned waves will be explained in the subsequent section.

After \(r_{D;NL}\) is found from Eq. (34) the coefficients of contribution can be updated using Eq. (35).

\[ C_{R;ij} = N_D P(R_{NL} > r_{D;NL}|h_i, t_j)f_{H, T_p}(h_i, t_j)w_{ij} \Delta h \Delta t \]  

(35)

Based on the values of these coefficients along the edges of the area in the scatter diagram under consideration more sea states are added. The iteration is stopped when the increase of \(r_{D;NL}\) is below a certain value. Focusing then on the dominating area, the probability, \(P(R_{NL} > r_{NL})\), that \(R_{NL}\) will be larger than \(r_{NL}\) may thus be approximated as

\[ P(R_{NL} > r_{NL}) = \sum_i \sum_j P(R_{NL} > r_{NL}|h_i, t_j)f_{H, T_p}(h_i, t_j)w_{ij} \Delta h \Delta t, \]  

(36)

again, for all combinations of \(i\) and \(j\) for which \(C_{R;ij} > \alpha\).

From what is presented in this section it will be clear that, in order for the proposed method to work, the accuracy with which \(P(R_{NL} > r_{NL}|h_i, t_j)\) can be predicted using response conditioned wave methods is of particular interest.

3. Case studies

3.1. General

In order to investigate the accuracy with which nonlinear short-term probability distributions can be predicted using response conditioned waves, the short-term cumulative probability distribution of the midship vertical hogging bending moment found in random irregular waves was compared with the one obtained using response conditioned waves. The investigations were done using a container vessel of newer design. The unique hull form of these vessels – large, flat and overhanging stern, and pronounced bow flare – can cause large ship motions relative to the water surface, which result in severe slamming impacts. The main particulars of the vessel and its conditions during the investigations are given in Table 1. The body plan is shown in Fig. 1.

Numerical and experimental investigations were carried out. In the former case two sea states were studied, namely the ones most contributing to the midship hogging moments with a mean return period of 20 years and 10 000 years. Experimentally only the latter sea state was investigated. Both sea states were found as the ones with the highest coefficient of contribution according to the iterative long-term analysis described in Section 2.2. More details regarding this analysis were presented by Drummen and Moan [13]. The parameters of the two sea states are presented in Table 2. The JONSWAP spectrum was used as wave spectrum. In all cases the vessel encountered head waves with 5 kn

<p>| Table 1 |</p>
<table>
<thead>
<tr>
<th>Main particulars of the vessel and its conditions during the investigations.</th>
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<tbody>
<tr>
<td><strong>Main particular</strong></td>
</tr>
<tr>
<td>Length overall</td>
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<tr>
<td>Length between perpendiculars</td>
</tr>
<tr>
<td>Beam</td>
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<td>Draft</td>
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<td>Trim</td>
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<td>Displacement</td>
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<tr>
<td>Speed</td>
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</tbody>
</table>
forward speed. The transfer function of the midship bending moment, for a forward speed of 5 kn, used for these analyses was presented by Drummen and Moan [13].

Drummen and Moan [13] also described a procedure to reduce the scatter diagram used for the long-term analyses to only include sea states for which the most likely wave, underlying the linear response with the desired return period, does not break. For a mean return period of 20 years it was found that this is the case for a scatter diagram that includes sea states with a zero crossing period, \( T_{z,0} \), of 9.5 s and higher. For a mean return period of 10 000 years this was 12.5 s and higher.

The reason for using the sea state most contributing to the hogging moment with a mean return period of 20 years is because this sea state is most relevant during the operational lifetime of the vessel. The reason that the sea state related to the response with a mean return period of 10 000 years was investigated is to consider realistic physical events if design for an annual failure probability of \( 10^{-4} \) is aimed at. Hogging is considered because it is the most relevant mode due to the nature of extreme waves (peaks are larger than troughs), and because it is the dominant mode for containerships. Moreover, the studies of Dietz [4] showed that the accuracy of the response conditioning methods was lower for hogging than for sagging moments.

### 3.2. Numerical analysis

The numerical simulations were performed with a program in which the nonlinear hydroelastic hybrid strip theory method presented by Wu and Moan [8] was implemented. The hybrid approach is a combination of the conventional direct load evaluation for a rigid body and the modal superposition for a flexible hull. It still accounts for the dynamic effects in the lowest few global flexible modes, but, to achieve improved computational efficiency, omits to evaluate the quasi-static responses in the higher

<table>
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<tr>
<th>Mean return period</th>
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<tr>
<td></td>
<td>20 years</td>
<td>10 000 years</td>
</tr>
<tr>
<td>( H_s )</td>
<td>15 m</td>
<td>19 m</td>
</tr>
<tr>
<td>( T_p )</td>
<td>13.28 s</td>
<td>15.89 s</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>5</td>
<td>4.75</td>
</tr>
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</table>

**Table 2**

Parameters of the most contributing sea states. \( H_s \), \( T_p \) and \( \gamma \) denote significant wave height, peak period and peakedness parameter, respectively.
The nonlinearities in the vertical motions and cross-sectional load effects are introduced in the form of a nonlinear vertical excitation force. In this way the relationship between the ship motions or the load effects and the excitation force can remain linear, while the excitation force is no longer linearly related to the incident wave. The total nonlinear excitation force consists of a linear part and a nonlinear modification part. The response is decomposed in the same manner. The linear part is evaluated using 2D or 2.5D strip theory. In this paper the former was used because of the low forward speed. The nonlinear modification part is obtained as the convolution of the linear impulse response function and the nonlinear modification force. The nonlinearities considered are due to the slamming impact force, the nonlinear incident wave force and the nonlinear hydrostatic restoring force. More details about the method were described by Wu and Moan [8,14], Wu and Hermundstad [15] and Drummen [16].

For the calculation of the global structural dynamic effect only the first global vertical flexural mode was taken into account. The number of wave components, \( N \), was equal to 1599 for all simulations. The mass and stiffness distributions were chosen equal to those measured for the experimental model. Details about these distributions were presented by Drummen [16].

Figs. 2 and 3 show the probability distributions of the midship hogging moments for the sea state contributing most to the hogging moment with a mean return period of 20 years. Both figures present the distributions as obtained by numerical simulations in random irregular waves (RIW), most likely response waves (MLRW) and conditional random response waves (CRRW). For comparison the figures also show the Rayleigh distributions, based on the zeroth spectral moment, Eq. (6).

The distributions denoted as ‘RIW’ were obtained by simulating the given sea state for a period of 240 h. Repetition of the wave signal was avoided by choosing the circular frequency randomly per frequency interval, as proposed by Wu and Moan [14,19]. Thus, in each frequency interval

\[
\Delta \omega_{e,n} = \Delta \omega_{e,n}^{\text{max}} - \Delta \omega_{e,n}^{\text{min}} \quad \text{where} \quad \Delta \omega_{e,n}^{\text{max}} = \Delta \omega_{e,n+1}^{\text{min}}
\]  

\( \omega_{e,n} \) had a random value. The avoidance of repetition was demonstrated in Fig. 4 of reference [19].

Results obtained by the MLRW method could be generated very fast, as this required only one short simulation per exceedance probability. Eight exceedance probabilities were investigated, \( 10^{-1}, 10^{-2} \ldots 10^{-8} \), which means that \( \zeta(t) \) was obtained from Eq. (11) for eight different linear responses, \( M_c \).

---

**Fig. 2.** Numerically obtained probability distributions of the midship vertical hogging bending moments for a rigid hull. \( H_k = 15 \text{ m}, \ T_p = 13.28 \text{ s} \) and \( \gamma = 5 \). RIW, MLRW and CRRW, respectively, denote random irregular wave, most likely response wave and conditional random response wave.
The latter were found from the exceedance probabilities using the Rayleigh distribution, see Eq. (38). The wave profiles were conditioned to give the desired linear response after 150 s. Each MLRW was simulated for 300 s. The chosen nonlinear response, \( r_{NL} \), was the maximum midship nonlinear vertical hogging bending moment in the time interval \( 148 < t < 152 \) s. In the presented results the sensitivity to the search width was, however, zero for the rigid hull and very small for the flexible hull, as long as the width was larger than approximately \( 149 < t < 151 \) s. In this way one \( r_{NL} \) was obtained for each \( M_c \), thereby linking the linear and the nonlinear response. The probability that the nonlinear response, \( R_{NL} \), will be larger than \( r_{NL} \) was obtained from

\[
P(R_{NL} > r_{NL} | h, t) = P(R_L > M_c | h, t) = \exp \left(- \frac{M_c^2}{2m_0} \right).
\] (38)

Note that this does not imply that \( P(R_{NL} > r_{NL} | h, t) \) follows the Rayleigh distribution, as \( r_{NL} \) will generally differ from \( M_c \).

Also the CRRWs were conditioned to give the desired linear response after 150 s, and the chosen nonlinear response was again the maximum midship nonlinear hogging moment in the time interval \( 148 < t < 152 \) s. But, also here the sensitivity to the search width was zero for the rigid hull and very small for the flexible hull, as long as the width was larger than approximately \( 149 < t < 151 \) s. Using these waves, two different sets of results were obtained. The first set, denoted ‘CRRW’, was found using the approach described by Dietz [4] and Dietz et al. [11]. For the sea state corresponding to the mean return period of 20 years 50 different response levels, \( M_c \), were investigated. For the one corresponding to the 10 000 years return period this was 60. For each response 100 simulations with CRRWs were performed. In this manner the conditional probability, \( P(R_{NL} > r_{NL} | r_L = M_c) \), that \( R_{NL} \) will be larger than \( r_{NL} \) was obtained. The unconditional probability was found by unconditioning with respect to the linear response

\[
P(R_{NL} > r_{NL} | h, t) = \int_0^\infty P(R_{NL} > r_{NL} | r_L = M_c, h, t) f_R(M_c) dM_c,
\] (39)

where \( f_R(M_c) \) denotes the Rayleigh probability density function of the response in the sea state characterised by a significant wave height \( h \) and peak period \( t \). This method, however, requires a large
number of runs, between 5000 and 6000 in this case. Experimental investigations by this method are thus not possible due to the efforts involved, as discussed by Drummen and Moan [13], and even numerically simulation times will be quite long. An overview of the simulated time per method is given in Table 3.

As a compromise between the MLRW method and a full CRRW analysis, 26 simulations using CRRWs were performed for eight exceedance probabilities, again $10^{-1}, 10^{-2}...10^{-8}$. Per exceedance probability the median of the 26 nonlinear moments was taken as the value presented in the figures. The probability that the nonlinear response, $R_{NL}$, will be larger than $r_{NL}$ was again found from Eq. (38), where $r_{NL}$ is now the median value described here. This approach is denoted as ‘CRRW (median)’.

Fig. 2 presents the results assuming a rigid hull girder. These results were found by low-pass filtering the response at a circular frequency of 1.75 rad/s. The full scale natural frequency of the lowest global flexural mode of the model was 3.5 rad/s [16]. Fig. 2 shows that moments obtained using response conditioned waves compare very well with those from random irregular waves. This is in agreement with results obtained by Dietz [4], and basically implies that the nonlinear peaks of the vertical bending moment correlate well with the linear peaks. Similar results for the flexible hull are given in Fig. 3. The figure shows that the distribution obtained using the MLRW method deviates from the one found in RIWs, and that the former has a peculiar drop starting at an exceedance probability of $10^{-4}$. This drop is explained as follows. The eight investigated MLRW profiles are scaled variations of the same wave, see Section 2.1. For low wave heights (high exceedance probabilities) there is almost no influence from slamming. As a result, the vertical bending moments for the flexible hull are approximately equal to those for the rigid hull. With increasing wave height (decreasing exceedance probability) slamming will start to occur around $t = 150$ s, increasing the maximum moment for the flexible hull compared to the rigid hull. As the wave height increases even further, slamming events will also occur at an earlier stage, so before $t = 150$ s. The whipping responses resulting from these slamming events will interfere with those from the slamming event around $t = 150$ s. As a result the maximum response at $t = 150$ s is decreased in this case. Since the results denoted as ‘CRRW (median)’ were based on 26 CRRWs, this method is less affected by the shape of one particular wave profile. The background wave introduces a randomness that prevents this from happening. For exceedance probabilities equal to $10^{-3}, 10^{-4}$ and $10^{-5}$, bending moments obtained from the CRRW (median) approach are, respectively, 17%, 23% and 25% lower than those found in RIWs. Exceedance probabilities between $10^{-3}$ and $10^{-5}$ were those relevant for the calculation of the long-term maximum value. Finally, Fig. 3 shows that the vertical bending moments obtained from a full CRRW analysis agree very well with those found in RIWs. But, as mentioned before, this method is computationally approximately as expensive as the traditional analysis in RIWs.

Similar probability distributions for the sea state most contributing to the vertical hogging moment with a mean return period of 10 000 years are shown in Figs. 4 and 5. The former figure again confirms the agreement between results from response conditioned and random irregular waves, for a rigid hull. Fig. 5 again shows that there is a good agreement between the hogging moments obtained from a full CRRW analysis and those found in RIWs. Bending moments found from the CRRW (median) approach are 17%, 21% and 20% lower than those found in RIWs for exceedance probabilities, respectively, equal to $10^{-3}, 10^{-4}$ and $10^{-5}$.

From the results presented in this section it is clear that the MLRW method can very efficiently be used to accurately determine the nonlinear short-term probability distributions of midship hogging

<table>
<thead>
<tr>
<th>Method</th>
<th>Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>RIW</td>
<td>864 000</td>
</tr>
<tr>
<td>MLRW</td>
<td>2 400</td>
</tr>
<tr>
<td>CRRW (median)</td>
<td>62 400</td>
</tr>
<tr>
<td>CRRW</td>
<td>1 500 000</td>
</tr>
<tr>
<td></td>
<td>1 800 000</td>
</tr>
</tbody>
</table>

* For the response with a mean return period of 20 years.

* For the response with a mean return period of 10 000 years.
moments in rigid hulls, in the case of low forward speed. The method can thus also efficiently and accurately be used for the long-term analysis proposed in Section 2.2. Here it should be noted that, as stated in Section 2.1, the MLRWs are independent of the significant wave height. Part, or all of the MLRW results for one sea state could therefore possibly be used in order to determine the short-term probability distributions in other sea states.

For the flexible hull it may be concluded that using the MLRW method in order to obtain the long-term extreme response can lead to an under-prediction of this response. In order to still benefit from

Fig. 4. Numerically obtained probability distributions of the midship vertical hogging bending moments for a rigid hull. $H_s = 19$ m, $T_p = 15.89$ s and $\gamma = 4.75$. RIW, MLRW and CRRW, respectively, denote random irregular wave, most likely response wave and conditional random response wave.

Fig. 5. Numerically obtained probability distributions of the midship vertical hogging bending moments for a flexible hull. $H_s = 19$ m, $T_p = 15.89$ s and $\gamma = 4.75$. RIW, MLRW and CRRW, respectively, denote random irregular wave, most likely response wave and conditional random response wave.
the efficiency of the MLRW method but improve the accuracy in these cases, Dietz [4] proposed to use the method in combination with a correction factor. This correction factor, which in most cases will depend on the exceedance probability, could then be determined by comparing MLRW results with those from brute force or CRRW simulations, for one sea state in the dominant area of the scatter diagram. For the other sea states in this area only MLRW results are obtained and used in the long-term analysis with the correction factors. From the results presented here it is however clear that this approach can lead to unreliable results because the complete probability distribution obtained for a sea state with this method is based on scaled variations of a single wave profile. Rather it is recommended to use the CRRW (median) method in combination with a correction factor as an accurate and efficient approach for obtaining the nonlinear long-term extreme responses of flexible hulls. Similarly as described above the correction factors are then determined by comparing CRRW (median) results with those from brute force or CRRW simulations for, say, the most contributing sea state. For the other sea states in the dominant area of the scatter diagram only CRRW (median) results are obtained and used in the long-term analysis with the correction factors.

From Fig. 3 it can be seen that the correction factors for obtaining the response with a mean return period of 20 years are 1.21, 1.30 and 1.33 for exceedance probabilities, respectively, equal to $10^{-3}$, $10^{-4}$ and $10^{-5}$. For the 10 000 years return period (Fig. 5) these values are 1.21, 1.27 and 1.24, respectively. The sea states part of the dominant area of the scatter diagram for the response with the 10 000 years return period were given by Drummen [16].

The assumption that the correction factors do not change within the dominant area of the scatter diagram is very reasonable as RIW and CRRW (median) results are affected in a similar manner by the small changes in the sea state characteristics within this area. In case multiple maxima in the scatter diagram need to be included in the long-term analysis [6] the correction factors should preferably be determined for each of the subareas.

### 3.3. Experimental investigation

The model tests were performed in the towing tank at the Marine Technology Centre in Trondheim. The tank is 260 m long, 10.5 m wide and between 5.6 m and 10 m deep. The double flap wave maker is able to produce both regular and irregular waves. The model was built using a scale of 1:45, resulting in a length between perpendiculars of 6.24 m. In order to account for the global hydroelastic effect in the experiments, the model was made of four rigid segments connected by three rotational springs. It could thus mimic the first three global vertical flexural modes of the full scale vessel. A picture of the model is shown in Fig. 6.

As mentioned in Section 3.1, the short-term probability distributions of the midship vertical hogging bending moments were obtained for the sea state most contributing to the hogging moment with a mean return period of 10 000 years. Four exceedance probabilities were investigated, $10^{-2}$, $10^{-3}$, $10^{-4}$ and $10^{-5}$. Details about the experimental setup and the test program were described by Drummen and Moan [13] and Drummen [16]. The latter reference also presented the challenges involved in using response conditioned waves experimentally in combination with a model with forward speed.

The experimental results are given in Figs. 7 and 8. It should be noted that the wave height measured by a wave probe 5 m in front of the wave maker was lower than the requested height at this location. Due to the nature of the response conditioned waves and the good repeatability [13], this however only influenced the absolute value of the response and did not influence the comparison between the random irregular and response conditioned waves. The transformation of the wave profile from the target location to that of the wave maker, described in Section 2.1, was done using Airy wave theory.

The short-term probability distribution of the response in random irregular waves was obtained by running the model in the given sea state for a period of one day, full scale. As mentioned above, the results by application of the MLRW method could be generated very fast, as this required only one run per exceedance probability. However, for sake of uncertainty each most likely response wave was run six times. The values presented in the figures represent the average of the six responses obtained for each particular exceedance probability. The dispersion of the results was small, as indicated by Drummen and Moan [13]. As also mentioned above, a full experimental CRRW analysis was not possible. In this case 10 CRRWs were run per exceedance probability, as a compromise between the
MLRW method and a full CRRW analysis. The figures show the median value resulting from this. For both the MLRW and the CRRW (median) methods the nonlinear responses and their exceedance probabilities were obtained in the same way as described in the previous section.

**Fig. 7** presents the short-term probability distribution of the midship vertical hogging bending moment assuming a rigid hull. The figure shows that the bending moments obtained in response

![Fig. 6. Model and towing carriage.](image)

![Fig. 7. Experimentally obtained probability distributions of the midship vertical hogging bending moments for a rigid hull.](image)

$H_s = 19$ m, $T_p = 15.89$ s and $\gamma = 4.75$ (requested). RIW, MLRW and CRRW, respectively, denote random irregular wave, most likely response wave and conditional random response wave.
conditioned waves compare well with those found in random irregular waves. Similar results for the flexible hull are presented in Fig. 8. Here it can be seen that the vertical bending moments obtained using MLRWs are approximately 15% lower than those found using RIWs. Results from CRRWs compare better with RIW results, although the deviation between the two increases with an increasing response.

3.4. Comparison between numerical and experimental results

Figs. 4 and 7, and Figs. 5 and 8 show a considerable difference between the vertical bending moments obtained numerically and experimentally. For the rigid hull, the calculated hogging moments are approximately 30% higher than the corresponding measured ones for an exceedance probability of $10^{-4}$. For the flexible hull this is about 100%. One important source for this difference is that, as described in the previous section, the wave height measured in the tank was lower than the requested height which was used in the simulations. Another source are the nonlinear wave interactions in the towing tank. During its propagation from the wave maker to the model, the wave was affected by these interactions. This was not accounted for in the numerical method since Airy wave theory was used. Thus part of the differences observed between Figs. 4 and 7, and Figs. 5 and 8 are related to incident waves which were not equal.

In order to better evaluate the accuracy of the numerical method, the calculations were repeated using the incident wave elevation the model experienced in the tank as input. However, the wave elevation measured at the centre of gravity of the model during the tests was disturbed by the model. Therefore, the MLRWs were repeated without the model in the tank. In this way the wave elevation at the location of the centre of gravity of the model could be measured, without the model disturbing the flow field. This could be done since Drummen and Moan [13] concluded that the waves repeated very well in the towing tank at the Marine Technology Centre in Trondheim. The comparisons between the experimentally and the numerically obtained vertical bending moments are shown in Figs. 9 and 10, and were done for the MLRW which led to a response with an exceedance probability of $10^{-4}$ in the sea state contributing most to the hogging moment with a mean return period of 10 000 years. Experimentally, all wave profiles were conditioned to give the desired response at approximately $t = 600$ s [13]. Sagging is positive in the figures.
Fig. 9 shows an almost perfect agreement between the measured and the calculated vertical bending moments. It also shows that the response of the vessel is not symmetric around the time instant used for conditioning. This is not in line with the remark made in Section 2.1, and is partly related to nonlinearities in the response, and partly to nonlinear wave interactions in the towing tank. The agreement is less around $t = 600$ s when the hull is considered flexible, Fig. 10. In this area the numerical method over-predicts the whipping response. The maximum hogging bending moment obtained using the numerical method is approximately 30% higher than the one found during the experiments. This is in line with the findings presented earlier by Drummen et al. [17,18]. An over-prediction of the high frequency component might have been caused by a too low structural damping in the numerical method. To avoid this problem, however, the structural damping of the two-node flexural mode in the numerical method was taken equal to the one measured on the test model, 0.65%
Moreover, Drummen et al. [17,18], who compared results from the same model tests with those from the same numerical method, showed that the inaccuracies in the high frequency component could not be explained by differences in damping. Furthermore, the fact that the calculated bending moment is too high in the short period immediately following the impact implies also here that damping is not the problem, since its influence during this period is limited due to a low damping ratio and a limited number of cycles. The over-prediction should more likely, at least partly, be attributed to the fact that 3D effects are not accounted for in the 2D slamming force calculation.

4. Conclusions

In this paper we presented the results of a numerical and an experimental investigation into the application of response conditioned waves for long-term nonlinear analyses. The proposed long-term analysis starts with the coefficient of contribution method, in which the extreme response is determined by considering only the few most important sea states. Subsequently, response conditioned wave methods are used to efficiently obtain the nonlinear short-term probability distributions of the vessel responses in these sea states. The accuracy of response conditioned wave methods for this purpose was investigated by comparing the short-term distributions obtained from random irregular waves (RIWs) with those from response conditioned waves. Both the most likely response wave (MLRW) and the conditional random response wave (CRRW) methods were investigated. The studies were performed using a container vessel, with a length between perpendiculars of 281 m. The calculations were done with a nonlinear hydroelastic hybrid strip theory method. The model tests were carried out in the towing tank at the Marine Technology Centre in Trondheim.

The comparisons showed that the MLRW method can be used very efficiently to accurately predict the nonlinear short-term probability distributions for rigid hulls, in the case of low forward speed. The method can thus also efficiently and accurately be used for the proposed long-term analysis of these hulls. When the hull girder is flexible, the MLRW method can no longer be used to accurately obtain the nonlinear short-term probability distributions. It was also found that performing the long-term analysis using the MLRW method in combination with a correction factor, as proposed by Dietz [4] can lead to unreliable results because the complete probability distribution obtained with this method is based on scaled variations of a single wave profile.

From numerically performing a full CRRW analysis it was found that the vertical bending moments obtained from this analysis agreed very well with those found in RIWs, even for the flexible hull. The simulated time was however long. For this paper approximately 20 days of simulated time was used. This time can be reduced by only performing simulations in the response range of interest, as well as by simulating only until several seconds after the conditioned event. It should, however, be noted that each single simulation using CRRWs requires a certain duration to eliminate the effects due to initial conditions. The required time for finding nonlinear short-term probability distributions from a full CRRW analysis will therefore in general be of the same order of magnitude as the time needed for a conventional analysis in RIWs, typically 100–150 h according to Wu and Moan [19].

As a compromise between the MLRW method and a full CRRW analysis, 26 numerical simulations using CRRWs were performed for eight different exceedance probabilities, and 10 CRRWs were run experimentally for four exceedance probabilities. The nonlinear response was chosen as the median value of the, respectively, 26 and 10 responses following from the analyses. Contrary to the MLRW method, this method depends on more than one wave profile. Furthermore, it requires considerably less time than a full CRRW analysis. It is therefore recommended to use the CRRW (median) method instead of the MLRW method in combination with a correction factor as an accurate and efficient approach for obtaining the nonlinear long-term extreme for flexible hulls. For short-term exceedance probabilities between $10^{-3}$ and $10^{-5}$ it was found that this correction factor varies approximately between 1.2 and 1.3.

A comparison between the time series of the measured and the calculated midship bending moments due to the same MLRW showed an almost perfect agreement when the hull was assumed rigid. For the flexible hull, the maximum hogging moment obtained using the numerical method was approximately 30% higher than the one found during the experiments. This is at least partly attributed to the fact that 3D effects are not accounted for in the 2D slamming force calculation.
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References