Chapter 4: Ultimate Strength of Plate- and Box-Girders

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4. PLATE- AND BOX-GIRDERS

4.1 General
Plate- and box-girders are extensively used in decks of drilling rigs, and as transverse and longitudinal frames of ships. They have also proved to be very efficient in the design of bridges.

4.1.1 Plate-Girders
Typical loads acting on a plate girder are shown in Figure 4.1,

![Figure 4.1 Loads on a Plate-Girder.](image)

The critical load and the associated buckling modes depend on the ratio of different load components: axial force, shear force, and bending moment. The geometrical parameters of importance are the web thickness, the flange area, and the location of secondary stiffeners and brackets. Depending on these quantities, the local buckling of web plate may be critical, (Figure 4.2), or, the simultaneous buckling of web plate, secondary stiffeners, and top flange, (Figure 4.3).

Plate girders subjected to shear, possess considerable strength reserves in the post-buckling region. This is due to tension fields developing in the web plate such that the load carrying effect changes. This is considered in Section 4.2.
4.1.2 Box Girders

The geometry and loading of a box girder is shown in Figure 4.4. Due to significant shear stresses in the flanges, the axial stress normally varies over the flange width. This effect is referred to as shear lag. The shear stresses in the flange may be due to bending as well as torsion.

For the compression flange, several failure modes exist, namely:

- local buckling of plating between stiffeners, possibly in combination with tripping of stiffeners
- local buckling of flanges close to the webs where the axial stress is maximum
- panel buckling between two transverse frames
- overall buckling of the entire flange with transverse frames

The latter mode may be eliminated by designing the transverse frame sufficiently strong.

At the moment of buckling in the flange plating adjacent to the web, the mid-zone is still not fully utilized. This is due to the shear lag distribution over the flange. Of crucial importance for the ultimate capacity, is the possibility of load shedding in the external zones until the internal zone becomes fully utilized.
4.2 Shear Capacity of Plate-Girders

4.2.1 Ultimate Strength - Weak Flanges - (Basler's theory)
As mentioned above, plate girders may possess significant reserve strength beyond the critical stress. The load carrying capacity in the post-critical range is predominantly related to the geometry of the structure, slenderness, material properties, and boundary conditions.

\[ \tau_k = \frac{\pi^2 E}{12(1-\nu^2)\left(\frac{h}{b}\right)^2} k \]  
(4.1)

where the buckling coefficient, \( k \), for simply supported edges is given by,

\[ k = 5.34 + \frac{4}{(a/b)^2} \quad \text{for} \quad a/b \geq 1 \]  
(4.2)
For plates with low slenderness, the elastic critical stress has to be modified for plasticity. However, webs of plate girders are typically very slender with $b/h$ ratio up to 300. For $b/h = 300$, there is obtained

$$
\bar{\lambda} = 2.65 \text{ for } \sigma_y = 240 \text{MPa} \Rightarrow \frac{\tau_{cr}}{\tau_y} = \frac{1}{\sqrt{1 + \bar{\lambda}^2}} \approx \frac{1}{\lambda^2} = 0.14 \quad (4.4)
$$

Tests have shown that such webs have considerable load carrying capacity beyond $\tau_{cr}$ due to stress redistribution in the web plate as indicated in Figure 4.6. As a result of the increase in the length of one of the diagonals, a tension band develops. It is assumed that the stress state can be splitted into two parts:

- a.) The stresses at the initiation of buckling, $\tau_{cr}$, with the principal stresses $\sigma_1 = \tau_{cr}$, $\sigma_2 = -\tau_{cr}$. The principal stresses are orientated at $45^\circ$ to the x-axis, and represent the beam effect. The associated shear force is denoted by $V_\tau$.
- b.) The additional stresses, caused by the tension band.

When the shear force is increased beyond the critical value, with respect to buckling, the web can not take increased pressure in the $\sigma_2$-direction. The compression stress, $\sigma_2$, remains constant and equal to minus $\tau_{cr}$. In the tension direction, there is no buckling problem and the tension stress may increase.

The tension stress has to be anchored in the top flange and the vertical stiffeners. First, it is assumed that the flanges are so flexible that the tension band can only be anchored by the vertical stiffeners as shown in Figure 4.8.
The width of the tension band is given by,

\[ s = b \cos \phi - a \sin \phi \]  \hspace{1cm} (4.5)

The tension band component becomes

\[ V_d = \sigma_d bh \sin \phi \]

\[ = \sigma_d bh \left( \cos \phi - \frac{a}{b} \sin \phi \right) \sin \phi \]  \hspace{1cm} (4.6)

which can also be written as,

\[ V_d = \sigma_d bh \frac{\tan \phi}{\sqrt{1 + \tan^2 \phi}} \left( 1 - \frac{a}{b} \tan \phi \right) \frac{1 - \frac{a}{b} \tan \phi}{1 + \tan \phi} \]

\[ = \sigma_d bh \tan \phi \left( 1 - \frac{a}{b} \tan \phi \right) \frac{1 - \frac{a}{b} \tan \phi}{1 + \tan \phi} \]  \hspace{1cm} (4.7)

It is natural to assume that the tension band inclination, \( \phi \), appears such that the shear force, \( V_d \), is maximized. Hence, we obtain,

\[ \frac{dV_d}{d \tan \phi} = \frac{\sigma_d bh}{(1 + \tan^2 \phi)} \left[ \left( 1 - \frac{2a}{b} \tan \phi \right) (1 + \tan^2 \phi) - 2 \tan \phi \left( \tan \phi - \frac{a}{b} \tan \phi \right) \right] = 0 \]  \hspace{1cm} (4.8)

which yields,

\[ \tan \phi = \sqrt{1 + \left( \frac{a}{b} \right)^2} - \frac{a}{b} = \tan \frac{\mu}{2} \quad \Rightarrow \quad \phi = \frac{\mu}{2} \]  \hspace{1cm} (4.9)

where,

\[ \tan \mu = \frac{b}{a} \]  \hspace{1cm} (4.10)

If Equation (4.9) is introduced to Equation (4.7), the tension band component becomes

\[ V_d = \sigma_d \frac{bh}{2} \left[ \tan \frac{\mu}{2} \right] \]  \hspace{1cm} (4.11)

It remains the work of determining the stress \( \sigma_d \). Two extreme cases is considered for this purpose. \( \sigma_d \) has to fulfil the following requirements:-

**Figure 4.8** Tension Band in Web Panel. No Anchoring in Flanges.
i.) For very slender webs \((b/h \rightarrow \infty)\), \(\tau_{cr}\) approaches zero. That is, the tension band predominates, hence,

\[
\frac{\sigma_d}{\sigma_y} = 1 \quad \text{for} \quad \frac{b}{h} \rightarrow \infty
\]  

(4.12)

ii.) For low slenderness \((b/h \rightarrow 0)\), the buckling stress approaches the yield stress, hence,

\[
\frac{\tau_d}{\tau_y} = 1 \quad \text{for} \quad \frac{b}{h} \rightarrow 0
\]  

(4.13)

For intermediate values, linear interaction is assumed. This yields,

\[
\frac{\sigma_d + \tau_{cr}}{\sigma_y \tau_y} = 1
\]  

(4.14)

or,

\[
\frac{\sigma_d}{\sigma_y} = 1 - \frac{\tau_{cr}}{\tau_y}
\]  

(4.15)

From this, the tension component can be written as,

\[
V_d = \frac{\sqrt{3}}{2} \left( 1 - \frac{\tau_{cr}}{\tau_y} \right) \tau_y b h \tan \frac{\mu}{2}
\]  

(4.16)

and the total force becomes,

\[
V_B = V_x + V_d = b h \left[ \tau_{cr} + \frac{\sqrt{3}}{2} \left( 1 - \frac{\tau_{cr}}{\tau_y} \right) \tau_y \tan \frac{\mu}{2} \right]
\]  

(4.17)

\[ \Rightarrow \quad V_B = V_p \left[ \frac{\tau_{cr}}{\tau_y} + \frac{\sqrt{3}}{2} \left( 1 - \frac{\tau_{cr}}{\tau_y} \right) \tan \frac{\mu}{2} \right]
\]  

(4.18)

where the plastic shear force, \(V_p\), is given by

\[
V_p = \tau_y b h
\]  

(4.19)

Figure 4.9 shows the post-buckling strength according to Basler's theory for two different aspect ratios \((a/b)\) for the web. It is seen that very slender webs possess significant post-buckling strength, especially for low aspect ratios. This can be achieved by applying many secondary stiffeners.

It is also seen that Basler's theory behaviour for weak flanges yields acceptable results as compared to the more accurate analysis ("\(\phi\) optimum") described in Section 4.2.2.

(Note: The curves in Figure 4.9 are based upon a reduced tension field width \((s_{\text{red}} = 0.9 \, s)\) in accordance with Eurocode 3 provisions)
4.2.2 Ultimate Strength - Strong Flanges

If the flanges are strong parts of the tension field, the tension band may be anchored by the flanges as indicated in Figure 4.10. It is assumed that the flanges can carry the tension band until hinges are developed in the flanges, and the plate girder starts to collapse in a mode as shown in Figure 4.10b. The distances $d_t$ and $d_b$ are found by means of the virtual work principle.

The internal virtual work in the top flange is given by,

$$\delta W = 2M_p \delta \theta$$  \hspace{1cm} (4.20)
where $M_{pt}$ is the plastic bending capacity of the top flange. This is influenced by the degree of axial stresses from bending in the top flange,

$$M_{pt} = \sigma_{yt} \frac{b_t t_t}{4} \left[ 1 - \left( \frac{N_t}{\sigma_y b_t t_t} \right)^2 \right]$$  \hspace{1cm} (4.21)

where $b_t$ is the width of the top flange, $t_t$ is the thickness of the top flange, and $N_t$ is the total axial force in the top flange. $\sigma_{yt}$ is the yield stress of the top flange.

The external virtual work is due to the tension band stresses. The component of the tension band stress, normal to the flange, is given by

$$\sigma'_d = \sigma_d \sin^2 \phi$$  \hspace{1cm} (4.22)

Hence, the corresponding external virtual work is

$$\delta W_e = (\sigma_d \sin^2 \phi_{w} d_t) \frac{d_t \delta \theta}{2}$$  \hspace{1cm} (4.23)

Equating the internal and the external work, we obtain

$$d_t = \frac{2}{\sin \phi} \sqrt{\frac{M_{pt}}{\sigma_d \phi w}} \leq a$$  \hspace{1cm} (4.24)

In a similar way for the bottom flange, we get

$$d_b = \frac{2}{\sin \phi} \sqrt{\frac{M_{pb}}{\sigma_d \phi w}} \leq a$$  \hspace{1cm} (4.25)

Failure of the tension band is now assumed to occur when the membrane stress, $\sigma_d$, combined with the stress state at buckling, reaches the yield stress, $\sigma_{yrs}$, of the web material.

![Figure 4.11 Stress State in a Web Plate.](image-url)
The two-dimensional stress state of the web is shown in Figure 4.11. Using Mohr’s transformation rule, the following stress components are obtained,

\[
\sigma_\phi = \tau_{cr} \sin 2\phi + \sigma_d \quad (4.26)
\]
\[
\sigma_{\phi \times \phi/2} = -\tau_{cr} \sin 2\phi \quad (4.27)
\]
\[
\tau_\phi = -\tau_{cr} \cos 2\phi \quad (4.28)
\]

Using von Mises’ yield criterion,

\[
\sigma_\phi^2 - \sigma_\phi \sigma_{\phi \times \phi/2} + \sigma_{\phi \times \phi/2}^2 + 3\tau_\phi^2 = \sigma_{yw}^2 \quad (4.29)
\]

we obtain the following for the membrane stress,

\[
\sigma_d = \sqrt{\sigma_{yw}^2 - 3\tau_{cr}^2 \left(1 - \frac{3}{4} \sin^2 2\phi\right) - \frac{3}{2} \tau_{cr} \sin 2\phi} \quad (4.30)
\]

It is observed from Figure 4.12 that the von Mises yield criterion yields a slightly higher stress than the simplified interaction assumed by Basler.

**Figure 4.12 Stress interaction in tension field**

The width of the tension band now becomes

\[
s = b \cos \phi - \left(a - d_i - d_r\right) \sin \phi \quad (4.31)
\]

The capacity of the web plate is given as follows
\[ V'_d = \sigma_d(\phi)bh \left[ \cot \phi - \frac{a}{b} + \frac{1}{b}(d_r(\phi) + d_s(\phi)) \right] \sin^2 \phi \] (4.32)

It is seen that both, the tension band stress as well as the lengths of the flanges, have become functions of the inclination, \( \phi \), of the tension band. It is therefore difficult to derive a closed form solution for the optimum value of this angle, \( \phi_{\text{optimum}} \), contrary to the case with weak flanges. It should rather be determined from varying \( \phi \) until \( V'_d \) is maximum. Experience shows that an approximate solution is at

\[ \phi = \frac{2}{3} \mu \] (4.33)

Hence, the total shear capacity becomes,

\[ \frac{V_b}{V_p} = \left[ \frac{\tau_{\sigma}}{\tau_y} + \frac{\sigma_d(\phi)}{\tau_y} \sin^2 \phi \left( \cot \phi - \frac{a}{b} + \frac{1}{b}(d_r(\phi) + d_s(\phi)) \right) \right] \] (4.34)

where \( \sigma_d \) is given by Equation (4.30), and \( d_r, d_s \) by Equations (4.24-25).

The ultimate capacity, according to Equation (4.34), is traced in Figure 4.9 for \( a/b=1 \). It is seen that the more sophisticated failure criterion for the web yields an additional reserve strength as compared with Basler’s theory.

A necessary condition for utilization of the post-buckling capacity of web plates is that the shear buckling load is not exceeded too frequently. This may give rise to low cycle fatigue in the same manner as discussed for plates.

### 4.2.3 Ultimate Strength - Combined Bending and Shear (Eurocode 3)

The shear capacity evaluations are based on the assumption that the web takes no bending moment. If the bending moment is large, this assumption is no longer valid. The influence of the bending moment can be categorized as follows:

i.) \( M < M_{fl} \)

where \( M_{fl} \) is plastic bending capacity of the girder with the flanges only. The shear capacity can be calculated according to the procedure described in Section 4.2.2. The bending moment influences the shear capacity only through only the reduction in plastic bending moment of the top flange and bottom flange, (Equation 4.21).

ii.) \( M_{fl} < M < M_P = \sigma_t \left( A_{fl}b + \frac{1}{4} I_{fl} b^2 \right) \)

where \( M_P \) is the plastic bending moment of the girder.

When the bending moment exceeds \( M_{fl} \), the flanges no longer contributes to supporting the tension field. In addition, the bending moment "consumes" parts of the load carrying
The resulting capacity in this range is therefore calculated according to the following formula

$$\frac{M - M_\beta}{M_p - M_\beta} + \left(1 - 2 \frac{V}{V_d} - 1\right)^2 = 1$$  \hspace{1cm} (4.35)

where $V_B$ is the shear capacity with no anchoring in flanges.

It appears that for $V < V_B$, the girder can be assumed to take on a fully plastic bending moment.

The interaction diagram is plotted Figure 4.13

![Interaction Diagram](image)

**Figure 4.13** Interaction diagram for combined bending and shear of plate girders (Eurocode 3)

The use of the tension filed concept for a plate girder with a given design is illustrated in Figure 4.14. The ultimate shear capacity is plotted versus the reduced slenderness ratio for shear buckling of the web for two different web aspect ratios a/b. The figure shows that the contribution to the anchoring of the tension field form the flanges can have a significant influence on the capacity.
Figure 4.14 Ultimate strength of a plate girder according to Eurocode 3.
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