Effective Width Concept

Effective width, long plate

\[
\frac{b_e}{b} = \frac{\sigma_{xm}}{\sigma_y} = \begin{cases} 
\frac{2}{\beta} - \frac{1}{\beta^2} & \beta \geq 1 \\
1 & \beta \leq 1 
\end{cases}
\]

Effective width, short plate

\[
a_e = \frac{\sigma_{ym}}{\sigma_y} = \frac{0.9}{\beta^2} + \frac{1.9}{\alpha\beta} \left(1 - \frac{0.9}{\beta^2}\right), \quad \alpha = \frac{a}{b}
\]

Plate slenderness

\[
\beta = \frac{b}{t} \sqrt{\frac{\sigma_y}{E}}
\]
Post buckling capacity of plates
Influence of boundary conditions

Boundary conditions depend on location of plate element

Classification example: Ship bottom

A: unrestrained
B: constrained
F: restrained
Post-buckling capacity of plates
Influence of in-plane restraint of long edges

Compressive stress \( \frac{\sigma_r}{\sigma_Y} = \frac{2\eta}{b - 2\eta} \)

Reduction factor \( R_r = 1 - \frac{\sigma_r}{\sigma_Y} E_i = 1 - \frac{2\eta}{b - 2\eta} \frac{2(\beta - 1)}{\beta} \), \( 1 < \beta < 2.5 \)

DnV Class. Note 30.1

\[ b_e = \frac{\sigma_{xu}}{\sigma_Y} = \frac{1.8}{\beta} - \frac{0.8}{\beta^2} \quad \beta \geq 1 \]

\[ \frac{b_e}{b} = 1 \quad \beta \leq 1 \]
Buckling of cylindrical shells

- Bifurcation point buckling: change from stable pre-buckling path to unstable post-buckling path
- Limit point buckling: due to imperfection sensitivity
Buckling modes

- **Shell buckling;**
  - Buckling of shell plating between stiffeners and frames.

- **Interframe shell buckling;**
  - Involves buckling of the longitudinal stiffener with associated shell plating.

- **Panel ring buckling;**
  - Buckling of rings with associated plate flange between longitudinal stiffeners.

- **General buckling;**
  - Involves bending of shell plating, longitudinal stiffeners as well as ring frames.

- **Torsional or local buckling of stiffeners and frames.**

- **Column buckling of the cylinder.**
Equilibrium equations

\[
\begin{align*}
  r \frac{\partial N_x}{\partial x} + \frac{\partial N_{\theta x}}{\partial \theta} &= 0 \\
  r \frac{\partial N_{x\theta}}{\partial x} + \frac{\partial N_{\theta}}{\partial \theta} &= 0 \\
  \nabla^4 w &= \frac{1}{D} \left( p + N_x \frac{\partial^2 w}{\partial x^2} + \frac{2}{r} N_{x\theta} \frac{\partial^2 w}{\partial x \partial \theta} + \frac{1}{r^2} N_{\theta} \frac{\partial^2 w}{\partial \theta^2} - \frac{1}{r} N_{\theta} \right)
\end{align*}
\]
Donnel’s equation

\[ \nabla^8 w = \frac{\nabla^4}{D} \left( N_x \frac{\partial^2 w}{\partial x^2} + \frac{2}{r} N_{x\theta} \frac{\partial^2 w}{\partial x \partial \theta} + \frac{1}{r^2} N_\theta \frac{\partial^2 w}{\partial \theta^2} \right) - \frac{Et}{Dr^2} \frac{\partial^4 w}{\partial x^4} \]
Stress analysis – beam theory
Stress analysis – ring stiffened cylinders

\[ \text{yielding} \]
Stress Distribution Over a Half-Ring Frame.

- Stress in unstiffened cylinder
- Stress at ring stiffener

Midway between rings
At ring stiffener

Location between ring $x/(l/2)$

Hoop stress

$\beta = 1$
$\beta = 2$
$\beta = 3$
Donnel’s equation

\[ \nabla^8 w = \frac{\nabla^4}{D} \left( N_x \frac{\partial^2 w}{\partial x^2} + \frac{2}{r} N_{x\theta} \frac{\partial^2 w}{\partial x \partial \theta} + \frac{1}{r^2} N_\theta \frac{\partial^2 w}{\partial \theta^2} \right) - \frac{Et}{Dr^2} \frac{\partial^4 w}{\partial x^4} \]

Axial compression

\[ D\nabla^8 w + \frac{Et}{r^2} \frac{\partial^4 w}{\partial x^4} + \frac{P}{2\pi r} \nabla^4 \left( \frac{\partial^2 w}{\partial x^2} \right) = 0 \]
Euler buckling stress – axial compression

\[ \sigma_{xE} = \frac{\pi^2 E}{12(1 - \nu^2)} \left( \frac{t}{l} \right)^2 \left[ \left( \frac{m^2 + \bar{n}^2}{m^2} \right)^2 + \frac{12 Z^2}{\pi^4} \frac{m^2}{(m^2 + \bar{n}^2)^2} \right] \]

\[ Z = \frac{l^2}{rt} \sqrt{(1 - \nu^2)} \]  
Batdorff parameter

\[ \sigma_{xE} = \frac{\pi^2 E}{12(1 - \nu^2)} \left( \frac{t}{l} \right)^2 \cdot \frac{4\sqrt{3}}{\pi^2} Z = 0.605 \frac{Et}{r} = \sigma_{cl} \]  
Minimum
The buckling coefficient for various modes

The graph shows the relationship between the Batdorf parameter $Z$ and the buckling coefficient for different modes $m,n$. The approximate buckling coefficient is indicated on the graph.
Buckling coefficients for axial compression

Batdorf parameter \( Z = \frac{l^2}{rt} \sqrt{(1-v^2)} \)

\( Z_s = \frac{s^2}{rt} \sqrt{(1-v^2)} \)
Approximate buckling coefficient – axial compression

\[ C_Z = \sqrt{1 + \frac{48Z^2}{\pi^4}} \]
Buckling coefficients for external pressure

\[ C_\theta = \left[ \frac{(1 + \bar{n}^2)^2}{n^2} + \frac{12Z^2}{\pi^4 \bar{n}^2(1 + \bar{n}^2)^2} \right] \min \]

\[ C_\theta^{(1)} = \frac{3Z}{\pi^2 \sqrt{1 - \nu^2}} \left( \frac{l}{r} \right) : n = 2 \]

\[ C_\theta^{(2)} = \frac{4\sqrt{6}}{3\pi} \sqrt{Z} : n = (6\pi^2 Z) \left( \frac{r}{l} \right) \]

\[ \bar{n} = \frac{n \ell}{\pi r} \]

\[ Z^{(1)} = \frac{32}{27} \pi^2 \left( 1 - \nu^2 \right) \left( \frac{l}{r} \right)^2 \]

Batdorf parameter \( Z = \frac{l^2}{rt \sqrt{1 - \nu^2}} \)
Buckling coefficient for torsion

\[ C_{Z\theta}^{(1)} = 5.34 \sqrt{1 + 0.0257 Z^{3/2}} \]

\[ C_{Z\theta}^{(2)} = \frac{2\sqrt{2}}{\pi^2} \frac{Z}{(1 - \nu^2)^{1/4}} \sqrt{\frac{\nu}{r}} \]

\[ C_{Z\theta}^{(3)} = 0.856 Z^{3/4} \]

\[ C_{Z\theta}^{(4)} = 5.34 \]

\[ Z^{(1)} = 80 \left( 1 - \nu^2 \right) \left( \frac{I}{t} \right)^2 \]

\[ Z^{(3)} \]

\[ C_{Z\theta}^{(3)}, \text{ WIDE PLATE} \]

\[ C_{Z\theta}^{(4)}, \text{ WIDE PLATE} \]

Batdorf parameter \( Z = \frac{I^2}{rt \sqrt{(1 - \nu^2)}} \)
Buckling of imperfect cylindrical shells

- Buckling load influenced notably by:
  - Nonlinear material
  - Shape imperfections
- Scatter in test results
- Design curve lower bound
Influence of shape imperfection on buckling coefficient

- Plates moderately sensitive to imperfections
- Shells sensitive to imperfections

\[ Z = \sqrt{1 - \nu^2} \frac{l^2}{rt} \]

\[ C = \psi \sqrt{1 + \left( \frac{\rho \xi}{\psi} \right)^2} \]

Classical theory

Imperfect shell

Shell factor

Imperfection factor

Plate factor
Curved panels – shell buckling

<table>
<thead>
<tr>
<th></th>
<th>$\psi$</th>
<th>$\xi$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Stress</td>
<td>4</td>
<td>$0.702Z_s$</td>
<td>$0.5\left(1 + \frac{r}{150t}\right)^{-0.5}$</td>
</tr>
<tr>
<td>Shear Stress</td>
<td>$5.34 + 4\left(\frac{s}{l}\right)^2$</td>
<td>$0.856\sqrt[3]{Z_s^{3/4}}$</td>
<td>0.6</td>
</tr>
<tr>
<td>Circumferencial Compression</td>
<td>$\left[1 + \left(\frac{s}{l}\right)^2\right]^2$</td>
<td>$1.04\frac{s}{l}\sqrt{Z_s}$</td>
<td>0.6</td>
</tr>
</tbody>
</table>
# Unstiffened cylindrical shell buckling

Buckling Coefficients For Unstiffened Cylindrical Shells, Mode a) Shell Buckling

<table>
<thead>
<tr>
<th>Force</th>
<th>$\psi$</th>
<th>$\xi$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Stress</td>
<td>1</td>
<td>0.702 $Z$</td>
<td>$0.5 \left(1 + \frac{r}{150t}\right)^{-0.5}$</td>
</tr>
<tr>
<td>Bending</td>
<td>1</td>
<td>0.702 $Z$</td>
<td>$0.5 \left(1 + \frac{r}{150t}\right)^{-0.5}$</td>
</tr>
<tr>
<td>Torsion and Shear force</td>
<td>5.34</td>
<td>$0.856Z^{3/4}$</td>
<td>0.6</td>
</tr>
<tr>
<td>Lateral Pressure</td>
<td>4</td>
<td>$1.04\sqrt{Z}$</td>
<td>0.6</td>
</tr>
<tr>
<td>Hydrostatic Pressure</td>
<td>2</td>
<td>$1.04\sqrt{Z}$</td>
<td>0.6</td>
</tr>
</tbody>
</table>