Implementing unsteady friction in Pressure-Time measurements

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Background

- Collaboration between LTU and NTNU
- Experiments on the pressure time method
Motivation

• A Reynolds number or initial velocity dependence in the error of the estimated flow rate observed in laboratory experiments and numerical simulations.
Motivation

• The Gibson method does not consider unsteady friction in the calculations
• Deceleration is transient
  → Transients cause unsteady friction
  → Source of error in the flow estimates?
Objective

Investigate if established models for unsteady friction could improve flow estimates
Gibson’s method

- Measures the discharge
- Fast stop
- Water hammer
Gibson’s method

• Based on Newton’s 2nd law

\[ F = m \cdot a = \rho \cdot L \cdot A \cdot \frac{dV}{dt} \]

\[ F = \rho \cdot g \cdot h \cdot A \]

\[ \rho \cdot g \cdot \Delta h_{1-2} \cdot A = \rho \cdot \Delta L \cdot A \cdot \frac{dV}{dt} \]

\[ \frac{dQ}{dt} = \frac{g \cdot A \cdot \Delta h_{1-2}}{\Delta L} \]

\[ Q = \frac{g \cdot A}{L} \int_{\text{start}}^{\text{end}} \Delta h \cdot dt \]

\[ Q = \frac{g \cdot A}{L} \int_{\text{start}}^{\text{end}} (\Delta h + \xi) dt + q_{\text{leaq}} \]
The friction term in the Gibson method

- $\xi = f \cdot \left(\frac{L \cdot \rho \cdot V^2}{2 \cdot D}\right)$
- $f$ is assumed constant throughout the deceleration
  $\rightarrow \xi = C \cdot Q^2$
- $C$ is derived from the initial friction (i.e., before valve closure)

$\xi_{line}(t) = \xi_{start} \cdot \left(1 - \frac{Q(t)}{Q_{tot}}\right)^2$
The friction term in the Gibson method

- $\xi = C \cdot Q^2$ is only valid in rough pipes and at slow deceleration rates
Numerical investigation

- The numerical model shows similar results as in the experiments
- 1D Method of Characteristics simulations with Brunone’s friction model
Unsteady friction model

• Brunone’s friction model: 

\[ f = f_q + \frac{kD}{V|V|} \left( \frac{\partial V}{\partial t} - a \frac{\partial V}{\partial x} \right) \]

\[ f_q = \left( -1.8 \cdot \log \left( \frac{6.9}{Re} + \left( \frac{e/D}{3.7} \right)^{1.11} \right) \right)^{-2} \quad \text{(Re>2300)} \]

\[ C^* = \frac{7.41}{Re \log(14.3/Re^{0.05})} \quad \text{(Re>2300)} \]

\[ k = \frac{\sqrt{C^*}}{2} \]

\[ C^* = 0.00476 \]
Unsteady friction model

• The influence from each term in Brunone’s friction model was studied in numerical simulations

\[ f = f_q + \frac{kD}{V|V|} \left( \frac{\partial V}{\partial t} - \alpha \frac{\partial V}{\partial x} \right) \]

Quasi-steady
Convective acceleration
Temporal acceleration
New calculation procedure

Unsteady Gibson’s calculation

• Implement quasi-steady and temporal terms (convective term dismissed)

\[ f = f_q + \frac{kD}{V|V|} \left( \frac{\partial V}{\partial t} \right) \]

• Iterative procedure where k, f_q and V are updated at each time step
New calculation procedure

Preparations:

• Start assuming linear friction line from start to end of integration, for first estimate of $Q_0$

• Start iterative loop that runs until convergence criterion is satisfied
New calculation procedure

For each time step, find:

\[ V_i = \frac{Q_0}{A} - \frac{1}{\rho L} \int_0^i (\Delta P_i + \xi_i) \, dt \Rightarrow Re = \frac{\rho VD}{\mu} \frac{dV}{dt} \]

\[ f_q = \left( -1.8 \cdot \log \left( \left( \frac{6.9}{Re} \right) + \left( \frac{\varepsilon}{D} \right)^{1.11} \right) \right)^{-2} \]

\[ C^* = \frac{7.41}{Re \log(14.3/Re^{0.05})} \Rightarrow \]

\[ f = f_q + \frac{kD}{V|V|} \left( \frac{\partial V}{\partial t} \right) \Rightarrow \xi = f \cdot \left( \frac{L \cdot \rho \cdot V^2}{2 \cdot D} \right) \]
New calculation procedure

• Insert values into flow equation and calculate $Q_{\text{new}}$
• If convergence criterion is not satisfied → set $Q_{\text{new}}$ to $Q_0$ and start again.
# Results

## Specifications for the measurements

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe diameter</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Measurement cross section distances</td>
<td>6 and 9 m</td>
</tr>
<tr>
<td>Pressure</td>
<td>9.75 m w.c*</td>
</tr>
<tr>
<td>Valve closure time</td>
<td>~5 s</td>
</tr>
<tr>
<td>Investigated flow rates</td>
<td>~0.16, ~0.3 and ~0.4 m³/s</td>
</tr>
<tr>
<td>Corresponding Reynolds number</td>
<td>~0.65·10⁶, ~1.25·10⁶ and ~1.70·10⁶</td>
</tr>
</tbody>
</table>

![Graphs showing flow estimation error vs. Reynolds number](image-url)

**Note:** The graphs illustrate the flow estimation error (%) against the Reynolds number (x 10⁶) for both standard and unsteady conditions. The data points and error bars indicate the variability in the estimation error across different flow scenarios.
Conclusion

• Implementing unsteady friction models into the calculations improves the flow estimates
• Corrects both over- and underestimation
• Mathematically easy to implement
Further work

• Find a relationship for unsteady friction for more cases
  – Field tests against an accurate reference
    • Larger diameters, more roughness, etc.
  – Laboratory tests on different pipe diameters
Thank you for your attention