AN INTRODUCTION TO

THE CONCEPT OF EXERGY AND ENERGY QUALITY

by

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PREFACE

The main objective of this document is to serve as the reading text for the Exergy topic in the course Engineering Thermodynamics 1 provided by the Department of Energy and Process Engineering at the Norwegian University of Science and Technology, Trondheim, Norway. As such, it will conform as much as possible to the currently used text book by Moran and Shapiro [1] which covers the remaining topics of the course. This means that the nomenclature will be the same (or at least as close as possible), and references will be made to Sections of that text book. It will also in general be assumed that the reader of this document is familiar with the material of the first 6 Chapters in Moran and Shapiro, in particular the 1st and the 2nd Law of Thermodynamics and concepts or properties such as Internal Energy, Enthalpy and Entropy.

The text book by Moran and Shapiro <u>does</u> provide material on Exergy, in fact there is an entire Chapter 7 on that topic. It is felt, however, that this Chapter has too much focus on developing various Exergy equations, such as steady-state and dynamic Exergy balances for closed and open systems, flow exergy, etc. The development of these equations is done in a way that may confuse the non-experienced reader.

In addition, when used in an introductory course on Thermodynamics, it is more important to provide an understanding about how Exergy relates to Energy Quality, and how various efficiencies can be developed for Exergy and Energy. This can be done without going into detailed derivations of Exergy equations, thus there is a need for a simpler text on the topic, focusing more on the applications of the concept, and this is the background and main motivation behind this document.

This document is the 3rd version of the Introduction to Exergy and Energy Quality. The 1st version was written under considerable time pressure in October/November 2008 and updated in March 2009. Any suggestions for improvements are most welcome; the same applies to information about detected errors in the text as well as typographical errors.

1. INTRODUCTION

The term *Exergy* was used for the first time by Rant [2] in 1956, and refers to the Greek words *ex* (external) and *ergos* (work). Another term describing the same is Available Energy or simply *Availability*. The term Exergy also relates to *Ideal Work* as will be explained later, and Exergy Losses relate to *Lost Work*.

One of the challenges in Thermodynamics compared to Mechanics is the introduction of somewhat abstract entities (or properties) describing *pVT* systems, such as Internal Energy, Entropy and Exergy. In addition, there are special energy functions such as Enthalpy, Helmholtz energy and Gibbs (free) energy that are important in thermodynamic analysis but can be difficult to fully comprehend. While Enthalpy is important for flow processes (open systems) in Mechanical Engineering Thermodynamics, Helmholtz energy (to define equations of state) and Gibbs free energy (for physical and chemical equilibrium) are important in Chemical Engineering Thermodynamics.

Some text books introduce Internal Energy and Entropy as a way to be able to formulate the 1st and 2nd Laws of Thermodynamics:

- "Assuming there is a property called Internal Energy (symbol U or u in specific form); then the 1st Law of Thermodynamics can be formulated for closed and open systems". The dynamic *Energy balance* for an open system in its most general form is then:

$$\frac{dE_{cv}}{dt} = \frac{dE_k}{dt} + \frac{dE_p}{dt} + \frac{dU}{dt}$$

$$= \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i \right) - \sum_e \dot{m}_e \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e \right) \qquad (1)$$

- "Assuming there is a property called Entropy (symbol S or s in specific form); then the 2nd Law of Thermodynamics can be formulated for closed and open systems". The dynamic *Entropy balance* for an open system in its most general form is then:

$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i \cdot s_i - \sum_e \dot{m}_e \cdot s_e + \dot{\sigma}_{cv} \qquad (2)$$

Fortunately, other text books, such as for example Moran and Shapiro [1], make an effort trying to visualize what these properties really are. Common descriptions for Internal Energy and Entropy are:

- Internal Energy (U) can be viewed as the Kinetic and Potential Energy at the micro level of the system; i.e. for atoms and molecules. Kinetic Energy at this level can take the form of translation, vibration and rotation of the molecules, while Potential Energy can be related to vibrational and electrical energy of atoms within molecules (vapor or liquid) or crystals (solids) as well as chemical bonds, electron orbits, etc.

- Entropy (S) can be viewed as a measure of the disorder (or "chaos") in the system. More scientifically, it can be said to be a measure of the randomness of molecules in a system. In recent years, there has been a shift away from using the terms "order" and "disorder", to using energy dispersion and the systems inability to do work.

Exergy can also be described in a similar but somewhat more complicated way. Szargut [3] used the following statement to explain the term:

- "Exergy is the amount of work obtainable when some matter is brought to a state of thermodynamic *equilibrium* with the common components of its surrounding nature by means of *reversible* processes, involving interaction only with the above mentioned components of nature".

Since the term "reversible processes" is used in the definition, one could simply say that:

- "The Exergy of a system at a certain thermodynamic state is the *maximum* amount of work that can be obtained when the system moves from that particular state to a state of equilibrium with the surroundings".

This is the background why Exergy is related to Ideal Work. It should also be emphasized that there is a strong link between Exergy and Entropy since Entropy production (the term $\dot{\sigma}_{cv}$ in the Entropy balance, Equation 2) is equivalent to Exergy Loss (given by $T_0 \cdot \dot{\sigma}_{cv}$), which again is equivalent to Lost Work. Also notice that while Exergy is the ability to produce work, Entropy was previously described as the systems inability to do work.

Finally in this introduction, and as indicated in the very title of this document, Exergy is an indication of Energy Quality. Different energy forms have different quality (or different amounts of Exergy) in the sense that they have different capabilities to generate work. This is the main difference between the 1st and the 2nd Laws of Thermodynamics. The former states that energy is conserved and makes no distinction between energy forms, while the latter states that energy quality is destroyed, thus the different energy forms have different energy quality. The energy transformation processes in a system can only proceed from a higher quality form to a lower quality form unless there is some net input of energy quality (such as for example work) from the surroundings.

2.   THE ENERGY QUALITY OR EXERGY OF HEAT

Before deriving a general mathematical definition that relates Exergy to other state variables and state functions (i.e. "properties"), the following simple example of transforming heat to work is used to illustrate the concept of Exergy in its simplest form.

The Kelvin-Planck formulation of the 2nd Law of Thermodynamics states that net work can not be produced from a thermodynamically cyclic process if there is interaction (i.e. heat transfer) with only one thermal reservoir. The simplistic understanding of this is that heat can not be converted to work on a 1:1 basis (i.e. with 100% efficiency), see Figure 1.a. In fact, the mathematical representation of Kelvin-Planck ($W_{cycle} \leq 0$) indicates that no work can be produced, and equality ($W_{cycle} = 0$) only applies to the reversible case.
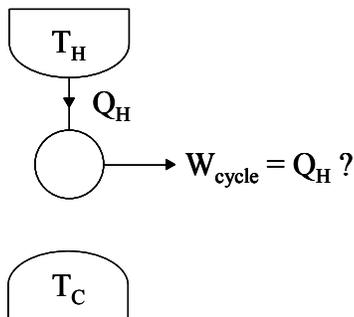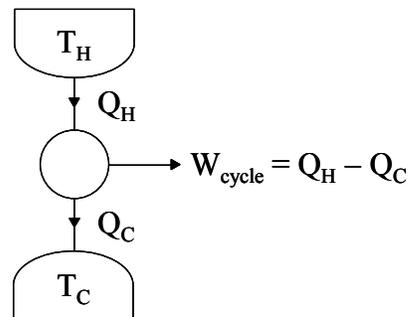

Figure 1.a   Cyclic process interacting   Figure 1.b   Cyclic process interacting
              with one thermal reservoir   with two thermal reservoirs

If, on the other hand, the thermodynamically cyclic process interacts with two thermal reservoirs (extracts heat Q_H from a reservoir with temperature T_H and delivers heat Q_C to a reservoir with temperature $T_C < T_H$), see Figure 1.b, then some work can be delivered by the system, ranging from zero to an upper bound given by the 2nd Law of Thermodynamics as shown below. For a cyclic process, the 1st Law of Thermodynamics reduces to:

$$\Delta U_{cycle} = Q_{cycle} - W_{cycle} = 0$$

$$\Rightarrow W_{cycle} = Q_{cycle} = Q_H - Q_C \tag{3}$$

Notice that the sign convention of Mechanical Engineering Thermodynamics (heat is positive if received by the system from the surroundings, and work is positive if developed by the system and applied to the surroundings) is not used here; all entities are regarded to be positive, and the arrows in Figures 1.a and 1.b indicate the direction of heat and work between the system (undergoing a cyclic process) and the surroundings.

The work that can be developed from an arrangement as the one in Figure 1.b is then constrained to be in the following range:

$$0 \leq W_{cycle} \leq Q_H \cdot \left(1 - \frac{T_C}{T_H}\right) \tag{4}$$

In Equation 4, one recognizes the maximum thermal efficiency to be that of the Carnot cycle, which is a reversible cyclic process with 2 adiabatic and 2 isothermal stages:

$$\eta_c = 1 - \frac{T_C}{T_H} \tag{5}$$

Rather than focusing on a cyclic process operating between two thermal reservoirs, consider an amount of heat Q at a temperature T. The amount of Exergy that this heat represents is stated by words in Section 1 to be the maximum amount of work (i.e. Ideal Work) that can be developed if equilibrium is established with the surroundings and only reversible processes are involved. A common definition of the *surroundings* is (despite variations in climate across the world) a large thermal reservoir with a fixed temperature $T_0 = 25°C$ (or ≈ 298 K) and a fixed pressure $p_0 = 1$ atm (or ≈ 1 bar).

Using Equation 4 and an analogy to the case in Figure 1.b, the surroundings act as the cold reservoir, while heat Q at a temperature T acts as the hot reservoir. The Exergy content of heat Q is then the maximum amount of work that can be extracted from this arrangement:

$$E_x = W_{max} = Q \cdot \left(1 - \frac{T_0}{T}\right) \tag{6}$$

Equation 6 illustrates one important fact regarding the Energy Quality of heat. For a given amount of heat Q, the amount of work that can be produced (and thus its Exergy or Energy Quality) increases with temperature. Thus, temperature is a quality parameter for heat.

A common graphical illustration of the quality (or Exergy) of heat is shown in Figure 2. The illustration is made in a Ts-diagram, since heat for internally reversible processes can be found from the following integral:

$$Q = \int T ds \tag{7}$$

Heat (as a form of energy) can be divided into one part that can produce work (its Exergy), and one part that can not be used to produce work, which is commonly referred to as

Anergy. For heat Q at temperature T, the following decomposition can be made with reference to the ambient temperature T_0 :

$$Energy \; = \; Exergy \; + \; Anergy \tag{8}$$

Unfortunately, Equation 8 only applies to heat, and this kind of Exergy will later be referred to as temperature based Exergy. When also considering pressure and composition, the simple relation in Equation 8 can no longer be used.
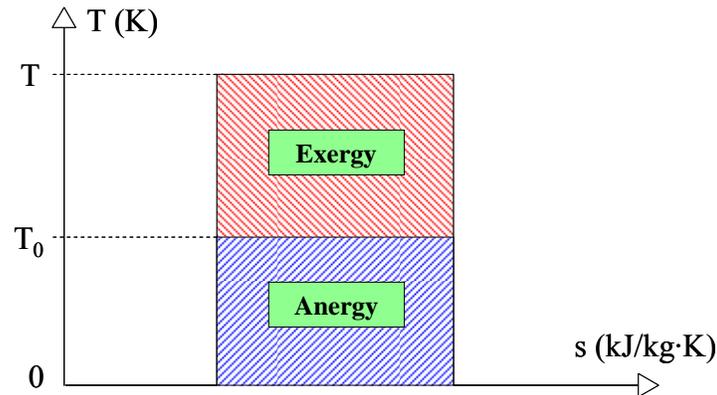

Figure 2   Decomposition of Energy (Heat) into Exergy and Anergy

3.   EXPRESSIONS FOR THERMO-MECHANICAL EXERGY

In the more general case, the System is more than an amount of heat, and the calculation of Exergy becomes a <u>bit</u> more complicated. Consider next a process stream with temperature T_1 and pressure p_1 with negligible kinetic and potential energy. The traditional *control volume* used by Moran and Shapiro [1] to discuss mass, energy and entropy balances for open systems (flow processes) is replaced by an idealized device used by Kotas [4] to develop expressions for Exergy. This device is shown in Figure 3.
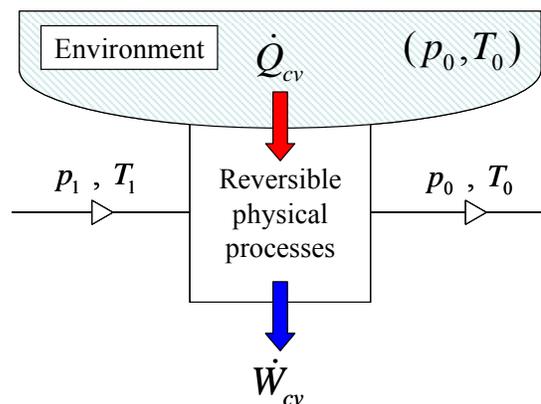

Figure 3   A reversible device (module) for determining thermo-mechanical Exergy

It should be emphasized that the heat transfer to/from the ideal reversible module in Figure 3 takes place at ambient temperature T_0, and that the work produced is the maximum possible (since the processes in the device are reversible) when the process stream at a

state or condition p_1, T_1 is brought to equilibrium with its natural surroundings, here referred to as the environment, which is at the thermodynamic state of p_0, T_0.

An energy balance for our system including the change of state for the stream passing through the ideal device in Figure 3, as well as the exchange of heat and work can be developed from Equation 1, assuming steady state and neglecting the changes in kinetic and potential energy for the process stream and the ideal device:

$$\frac{dE_{cv}}{dt} = 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \cdot (h_1 - h_0) \tag{9}$$

Similarly, the entropy balance assuming no irreversibilities can be developed from Equation 2:

$$\frac{dS_{cv}}{dt} = 0 = \frac{\dot{Q}_{cv}}{T_0} + \dot{m} \cdot (s_1 - s_0) \tag{10}$$

Equation 10 can be solved to establish an expression for specific heat transfer:

$$\frac{\dot{Q}_{cv}}{\dot{m}} = T_0 \cdot (s_0 - s_1) \tag{11}$$

Equation 9 can then be solved with respect to specific work (deliberately changing the sign of the enthalpy terms):

$$\frac{\dot{W}_{cv}}{\dot{m}} = T_0 \cdot (s_0 - s_1) - (h_0 - h_1) \tag{12}$$

Exergy was introduced in Section 1 as the maximum amount of work that can be obtained when the system moves from a particular thermodynamic state to a state of equilibrium with the surroundings. This change in thermodynamic state is exactly what is happening for the device in Figure 3. Since Exergy is related to the ability to produce work, it is obvious that Exergy as a property of the system (in this case the stream at condition p_1, T_1) will decrease when work is produced. Following the sign convention applied in Mechanical Engineering Thermodynamics, work is positive when following the direction of the arrow in Figure 3. The corresponding Exergy change will then have the opposite sign. In fact, the following general relation can be given:

$$\Delta E_x = -W_{ideal} \tag{13}$$

Equation (13) is more relevant when considering processes between two thermodynamic states for a system. Exergy can also be viewed as an "absolute" property and measured by the work obtained when the system moves from its current state to a state of equilibrium with the surroundings. The specific Exergy of the process stream entering the ideal device in Figure 3 can be derived from Equation 12 as follows:

$$e_x = T_0 \cdot (s_0 - s_1) - (h_0 - h_1) = (h_1 - h_0) - T_0 \cdot (s_1 - s_0) \tag{14}$$

The change in specific Exergy when a system undergoes a process from thermodynamic state 1 to state 2 is then given by the following expression (notice that the Enthalpy and Entropy functions at the surrounding conditions (h_0, s_0) cancel due to subtraction):

$$\Delta e_x = (e_{x,2} - e_{x,1}) = (h_2 - h_1) - T_0 \cdot (s_2 - s_1) = \Delta h - T_0 \cdot \Delta s \qquad (15)$$

Kotas [4] introduced the term *specific exergy function*, and it can be established by a simple rearrangement of Equation 15:

$$e_{x,2} - e_{x,1} = (h_2 - T_0 \cdot s_2) - (h_1 - T_0 \cdot s_1) \qquad (16)$$

The specific exergy function β and the corresponding total exergy function B are not important for this document, but they are included to show similarity with another energy function often used for physical and chemical equilibrium in Chemical Engineering Thermodynamics called Gibbs free energy G. These three functions are shown in Equation (17).

$$\begin{aligned} \beta &= h - T_0 \cdot s \\ B &= H - T_0 \cdot S \\ G &= H - T \cdot S \end{aligned} \qquad (17)$$

The situation illustrated by the device in Figure 3 only relates to pressure and temperature changes; the issue of chemical composition is not included. Further, since changes in kinetic and potential energy are ignored, the remaining exergy is only related to pressure and temperature and is referred to as *thermo-mechanical* Exergy. The change in total thermo-mechanical Exergy for a system is given by:

$$\Delta E_x^{(tm)} = \Delta H - T_0 \cdot \Delta S \qquad (18)$$

4.   CLASSIFICATION AND DECOMPOSITION OF EXERGY

The classification of Exergy can be done in a way that is similar to the classification of energy, with some exceptions, see Figure 4. Emphasis in the course Engineering Thermodynamics 1 is on thermo-mechanical Exergy.
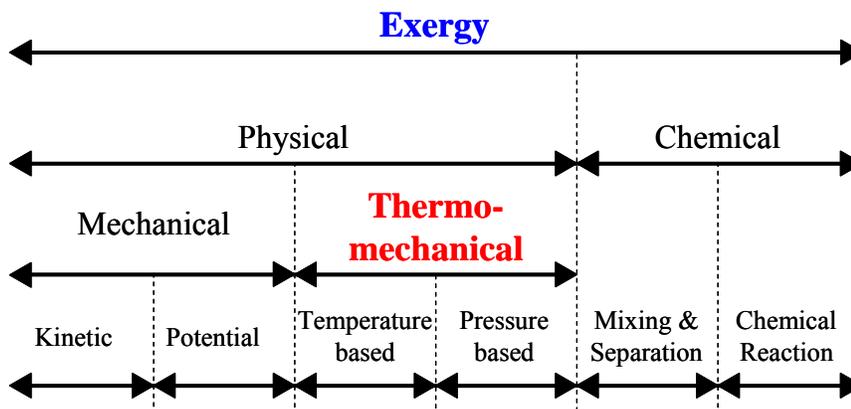

Figure 4   Classification of Exergy for *pVT* systems

As indicated in Figure 4, thermo-mechanical Exergy can be decomposed into a pressure based part and a temperature based part. This decomposition has no real fundamental significance, and is only done as a matter of convenience as will be described below. In fact, the decomposition is not even unique, which underlines its lack of a deeper meaning. Consider a system at thermodynamic state p, T that undergoes a set of reversible processes to end up at a thermodynamic state in equilibrium with the surroundings p_0, T_0, where equilibrium for pressure and temperature of course simply means equality.

The lack of uniqueness in the decomposition of thermo-mechanical Exergy is illustrated in Figure 5, where two logical ("extreme") paths (sets of processes) can be followed from the actual state to equilibrium with the surroundings. These paths are (a+b) and (c+d). In addition, both pressure and temperature could change simultaneously as indicated by path (e). In the literature, there is some common agreement, that the *temperature based* part of Exergy relates to first bringing the temperature of the system from T to T_0 while keeping the pressure constant at p. The *pressure based* part of Exergy then relates to the change in pressure from p to p_0 while keeping the temperature constant at T_0.

This decomposition is given by the path (a+b) in Figure 5. It should be emphasized that other decompositions, such as path (c+d), would give different results for the temperature based and the pressure based components. The total change in thermo-mechanical Exergy is, of course, independent of the selected path, since Exergy is a property of the system.
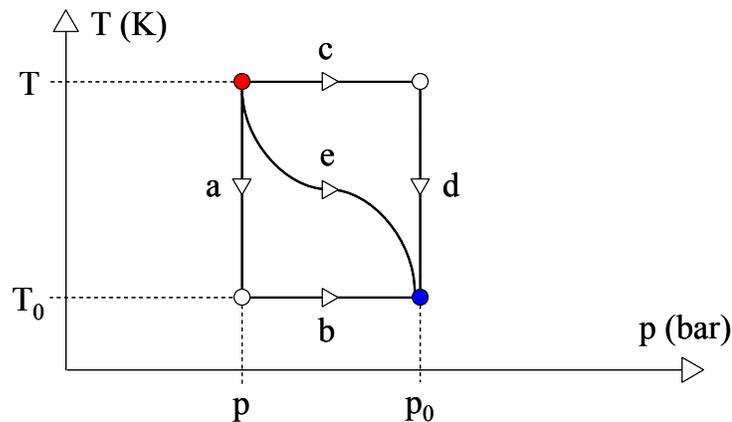

Figure 5   Decomposition of Thermo-mechanical Exergy

The decomposition of thermo-mechanical Exergy can be illustrated for the simple case of an ideal gas. Equation 14 shows that thermo-mechanical Exergy relates to Enthalpy and Entropy as repeated in Equation (19):

$$e_x^{(tm)} = (h - h_0) - T_0 \cdot (s - s_0) \tag{19}$$

By using the definition of temperature and pressure based Exergy given above in words, the following general expressions can be used to describe these two components of thermo-mechanical Exergy with reference to Figure 5:

$$e_x^{(T)} = \left[h(T, p) - h(T_0, p)\right] - T_0 \cdot \left[s(T, p) - s(T_0, p)\right] \tag{20}$$

$$e_x^{(p)} = \left[h(T_0, p) - h(T_0, p_0)\right] - T_0 \cdot \left[s(T_0, p) - s(T_0, p_0)\right] \tag{21}$$

For ideal gas, Enthalpy is only a function of temperature, while Entropy is a simple function of both pressure and temperature. Equations 22 and 23 show these ideal gas relations for Enthalpy and Entropy respectively.

$$dh = c_p(T) \cdot dT \tag{22}$$

$$ds = c_p(T) \cdot \frac{dT}{T} - R \cdot \frac{dp}{p} \tag{23}$$

A further simplification can be made by assuming constant specific heat capacity, c_p. Equations 22 and 23 can then be integrated from state 0 to state (T,p) with Enthalpy h and Entropy s as follows:

$$h - h_0 = c_p \cdot (T - T_0) \tag{24}$$

$$s - s_0 = c_p \cdot \ln\frac{T}{T_0} - R \cdot \ln\frac{p}{p_0} \tag{25}$$

By using the mathematical definitions for temperature based and pressure based Exergy given by Equations 20 and 21, the following expressions can be derived for the case of an ideal gas with constant specific heat capacities (and thus constant $k = c_p/c_v$):

$$e_x^{(T)} = c_p \cdot \left[T - T_0 \cdot \left(1 + \ln\frac{T}{T_0}\right)\right] \tag{26}$$

$$e_x^{(p)} = T_0 \cdot R \cdot \ln\frac{p}{p_0} = \frac{k-1}{k} \cdot c_p \cdot T_0 \cdot \ln\frac{p}{p_0} \tag{27}$$

Equations 26 and 27 show that temperature and pressure based Exergy can be manipulated individually by changing temperature and pressure respectively. Even more interesting is the fact that within the total thermo-mechanical Exergy, the two components can be traded-off against each other. This is deliberately exploited in subambient (or cryogenic) processes. A pressurized (gas) stream can be expanded to provide extra cooling and some power. In this operation, pressure based Exergy is converted into temperature based Exergy. Above ambient, the situation is completely different. A turbine (or expander) above ambient will take both pressure and temperature closer to ambient conditions, and as a result both pressure based and temperature based Exergy will be reduced. This is one explanation why the produced work (or power) is much larger above ambient than below ambient for the same pressure ratio. Another explanation, of course, is that expansion at higher temperatures provides more work since the gas volumes are larger.

In Figure 6, this difference above and below ambient is illustrated by an expander and an expression for the corresponding work developed in the two cases. Notice that no sign convention is used in Figure 6 and the effect on produced work is handled by logic. The change in pressure Exergy (a reduction) is the background for work being developed at all

in both cases. Above ambient, there is also a reduction in temperature based Exergy that adds to the work produced. Below ambient, however, the reduced temperature (increased distance to ambient temperature) <u>increases</u> the temperature based Exergy, and this results in a reduction of the work developed.
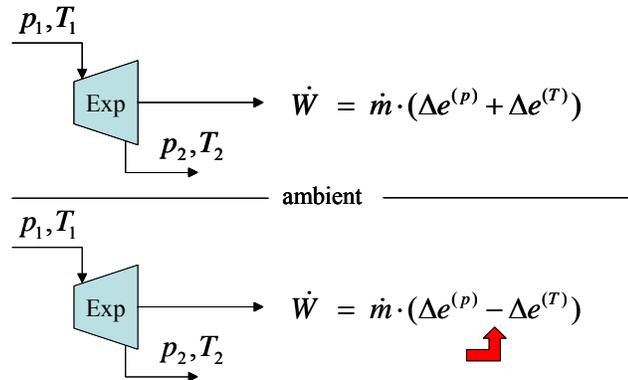

Figure 6   Turbine (expander) above and below ambient

In conclusion, above ambient the main objective for using an expander (not a valve) is to produce work, while the main objective below ambient is to produce cooling with work as a byproduct. For a gas based refrigeration cycle (referred to as an inverse Brayton cycle), an expander is required to be able to reduce the temperature to a point where the working fluid can absorb heat from the system to be cooled. Using a valve for a working fluid only operating in vapor phase will only marginally reduce the temperature, since a valve operates close to isenthalpic, and for an ideal gas, enthalpy only depends on temperature, not pressure. Even for real gases, enthalpy changes are moderate with changing pressure. Considering ideal gas and isentropic expansion/compression, Equation 28 can be derived (see Moran and Shapiro [1]) for the relationship between temperature and pressure:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} \tag{28}$$

Equation 28 can be used to calculate the outlet temperature from an expander or compressor when the inlet temperature is known as well as the pressure ratio.

<u>Example 1</u>:

Consider air as an ideal gas being expanded from 5 bar to 1 bar assuming reversible and adiabatic (i.e. isentropic) operation of the expander. Two cases will be analyzed, one case above ambient, where the ideal gas has an inlet temperature of 250ºC (523 K), and one case below ambient, where the ideal gas has an inlet temperature of -50ºC (223 K). Assume ambient temperature to be 25ºC (298 K) and ambient pressure to be 1 bar. In the calculations, air can be assumed to have the following properties:

$$c_p = 1.0 \; \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad , \quad k = 1.4 \quad , \quad R = \frac{\overline{R}}{M} = \frac{8.314}{28.97} = 0.287 \; \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

As indicated by Equation 13, the ideal work produced equals the change in Exergy with opposite sign. Temperature and pressure based Exergy can be obtained from Equations 26 and 27. The two cases will be referred to as case a (above ambient) and case b (below ambient). The corresponding states before (1) and after (2) expansion will be referred to as states 1a, 2a and 1b, 2b. The temperature of the ideal gas exiting the expander is found from Equation 28 for the two cases:

$$\text{Above ambient:} \quad T_{2a} = 330.2 \text{ K} \quad , \quad \text{Below ambient:} \quad T_{2b} = 140.8 \text{ K}$$

For the case <u>above</u> ambient, the following calculations can be made:

State 1a:
$$e_{x,1a}^{(T)} = 1.0 \cdot \left[523 - 298 \cdot \left(1 + \ln \frac{523}{298} \right) \right] = 57.38 \text{ kJ/kg}$$

$$e_{x,1a}^{(p)} = 298 \cdot 0.287 \cdot \ln \frac{5}{1} = 137.65 \text{ kJ/kg}$$

State 2a:
$$e_{x,2a}^{(T)} = 1.0 \cdot \left[330.2 - 298 \cdot \left(1 + \ln \frac{330.2}{298} \right) \right] = 1.62 \text{ kJ/kg}$$

$$e_{x,2a}^{(p)} = 298 \cdot 0.287 \cdot \ln \frac{1}{1} = 0 \text{ kJ/kg}$$

The maximum (ideal) work produced by the expander is then:

$$\left(\frac{\dot{W}_{ideal}}{\dot{m}} \right)_{above} = -\Delta e_x^{(T)} - \Delta e_x^{(p)} = -(1.62 - 57.38) - (0 - 137.65) = 137.65 + 55.76 = \underline{193.41 \text{ kJ/kg}}$$

For the case <u>below</u> ambient, the following calculations can be made:

State 1b:
$$e_{x,1b}^{(T)} = 1.0 \cdot \left[223 - 298 \cdot \left(1 + \ln \frac{223}{298} \right) \right] = 11.40 \text{ kJ/kg}$$

$$e_{x,1b}^{(p)} = 298 \cdot 0.287 \cdot \ln \frac{5}{1} = 137.65 \text{ kJ/kg}$$

State 2b:
$$e_{x,2b}^{(T)} = 1.0 \cdot \left[140.8 - 298 \cdot \left(1 + \ln \frac{140.8}{298} \right) \right] = 66.23 \text{ kJ/kg}$$

$$e_{x,2b}^{(p)} = 298 \cdot 0.287 \cdot \ln \frac{1}{1} = 0 \text{ kJ/kg}$$

The maximum (ideal) work produced by the expander is then:

$$\left(\frac{\dot{W}_{ideal}}{\dot{m}} \right)_{below} = -\Delta e_x^{(T)} - \Delta e_x^{(T)} = -(66.23 - 11.40) - (0 - 137.65) = 137.65 - 54.83 = \underline{82.82 \text{ kJ/kg}}$$

By expanding air as an ideal gas below ambient temperature, 137.65 kJ/kg of pressure based Exergy is transformed into 54.83 kJ/kg of temperature based Exergy and 82.82 kJ/kg of work. Above ambient, 137.65 kJ/kg of pressure based Exergy adds to 55.76

kJ/kg of temperature based Exergy to produce 193.41 kJ/kg of work, i.e. 133.5% more work (or a factor of 2.3) for the same pressure ratio!

5. EFFICIENCY MEASURES FOR ENERGY

A large number of efficiencies have been proposed in Thermodynamics and elsewhere to measure the quality of processes and their energy utilization. Great care should be taken in choosing such efficiencies to make sure the answers given are relevant for the questions asked. Another common requirement for efficiencies is that they are 0-1 normalized (i.e. they should preferably have values between 0 and 100%), since relative numbers are easier to interpret than absolute numbers. This section will review some of the most common efficiencies used in energy conversion processes, trying to highlight their advantages and disadvantages.

It is common in various process industries to measure the quality of individual processes by the *specific energy consumption* given by the ratio between energy consumption (in MJ, kWh, etc.) and materials production (in tons of product). This type of efficiency has the advantage that one can easily compare different plants having the same main product; however, there are a number of disadvantages as well. First, there is no indication what the best possible performance is, and secondly, these numbers do not have a meaning unless they also take into account the energy content of raw materials and products.

5.1 THERMODYNAMIC EFFICIENCIES

In Thermodynamics, the most obvious way to express efficiencies is to compare the real behavior of a process (or equipment) with the ideal behavior. The term ideal process is related to the absence of thermodynamic losses, referred to as irreversibilities. When reversible behavior is combined with the absence of unintended heat transfer (adiabatic), then the processes are referred to as isentropic.

Unfortunately, one cannot express these efficiencies by a single general equation. In Mechanical Engineering Thermodynamics there is a need to distinguish between power producing processes and power demanding processes. The corresponding efficiencies are given by Equations 29 (power producing processes) and 30 (power demanding processes):

$$\eta_{TD} = \frac{\text{Actual useful energy produced}}{\text{Ideal (max) useful energy produced}} \quad\quad (29)$$

$$\eta_{TD} = \frac{\text{Ideal (min) useful energy required}}{\text{Actual useful energy required}} \quad\quad (30)$$

Isentropic efficiencies for turbines and compressors are examples of such *thermodynamic efficiencies*. Consider the turbine and compressor in Figures 7 and 8 operating between an initial state (1) and a final state (2). Ideal processes would have a final state (2s) where "s" is an indication of isentropic behavior. This simply means that the entropy in the final state equals the entropy of the initial state. For a compressor, <u>more</u> work is needed for a real process than an ideal process. For a turbine, however, <u>less</u> work is produced in a real process than in an ideal process. This obvious difference is the reason why there is a need for two expressions for thermodynamic efficiency (Equations 29 and 30).
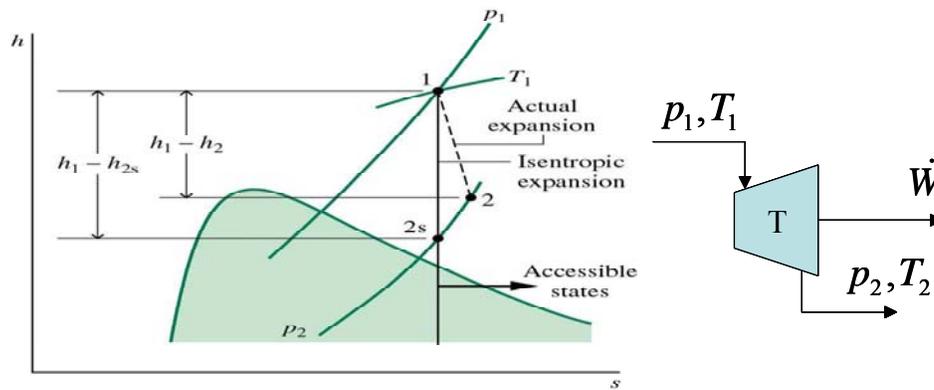
Figure 7   Isentropic Efficiency for a Turbine (Moran and Shapiro [1])
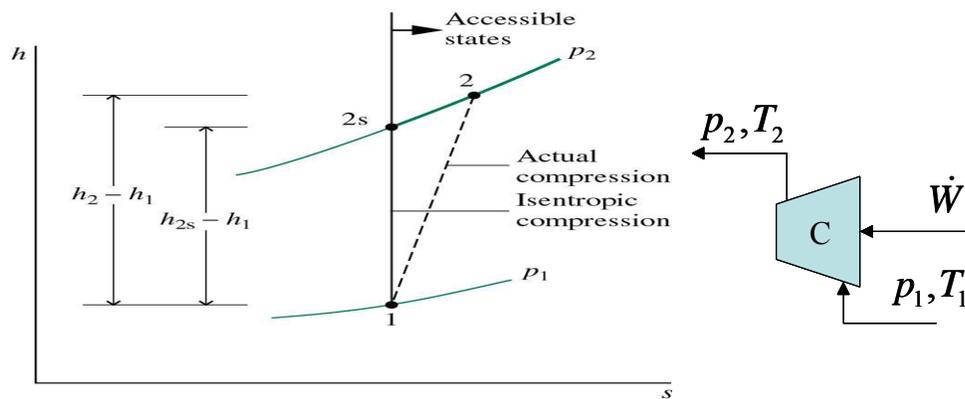

Figure 8   Isentropic Efficiency for a Compressor (Moran and Shapiro [1])

The mathematical expressions for thermodynamic efficiency, often referred to as *isentropic efficiency*, for turbines and compressors are given by Equations 31 and 32:

$$\eta_t = \frac{\left(\dot{W}_t / \dot{m}\right)_{real}}{\left(\dot{W}_t / \dot{m}\right)_{ideal}} = \frac{h_1 - h_2}{h_1 - h_{2s}} \tag{31}$$

$$\eta_c = \frac{\left(\dot{W}_c / \dot{m}\right)_{ideal}}{\left(\dot{W}_c / \dot{m}\right)_{real}} = \frac{h_{2s} - h_1}{h_2 - h_1} \tag{32}$$

Typical isentropic efficiencies for turbines are in the range 70-90%, while similar numbers for compressors are in the range 75-85%.

5.2 ENERGY EFFICIENCIES

The 1st Law of Thermodynamics states that even though Energy can be transformed, transferred and stored, it is always conserved. Nevertheless, different energy forms have different quality, and it makes sense to distinguish between useful energy and energy forms that can not be utilized. Friction, for example, causes energy "losses" in the sense that the heat developed because of friction is scattered and has a fairly low temperature, thus it can not easily be utilized.

A typical energy efficiency can then be expressed as follows:

$$\eta_E \;=\; \frac{\text{Useful Energy from the Process}}{\text{Useful Energy to the Process}} \tag{33}$$

This kind of efficiency is often used to measure the quality of steam turbines, gas turbines, combined cycle power stations, heat pumps and refrigerators. The common term used in Thermodynamics is *thermal efficiency*, even though focus is both on heat (thermal energy) and power (mechanical energy). Figure 9 shows power producing and power demanding thermodynamically cyclic processes operating between two thermal reservoirs.
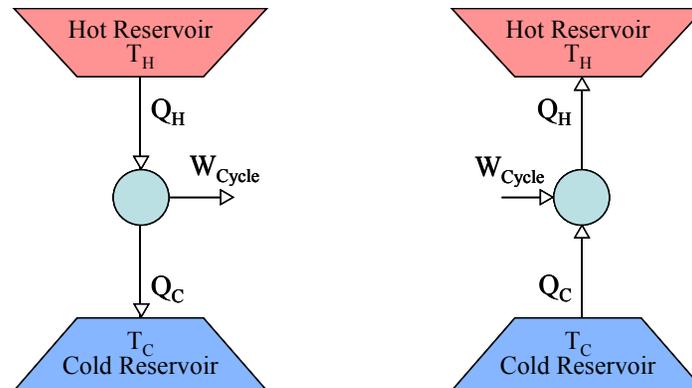

Figure 9   Conversion Processes for Power, Heating and Refrigeration

For power producing processes (left part of Figure 9), the heat that is transferred to the cold reservoir is normally not regarded as useful energy, since the cold reservoir quite often is air or cooling water. For a steam turbine, the effluent heat <u>could</u> be used for process heating, if Q_C is provided in the form of for example LP steam. The turbine will then be referred to as a back pressure or extracting turbine. The effluent heat could also be supplied to district heating systems or even fish farming.

The two situations with respect to what happens to the effluent heat in the left part of Figure 9 can be illustrated by providing two different thermal efficiencies. First, consider the case where the effluent heat is utilized:

$$\eta_E \;=\; \frac{W_{cycle} + Q_C}{Q_H} \tag{34}$$

If heat losses from the turbine can be neglected, then the thermal efficiency will approach 100%. This kind of efficiency then only deals with book-keeping, and it only detects possible heat losses from the process (or equipment). A thermal efficiency defined in this way fails to express the capability of the process to produce power, if that was the main objective, and it does not indicate how well the process performs relative to ideal (and reversible) processes. Secondly, consider the case where effluent heat has a temperature that is too low for the heat to be utilized. The thermal efficiency can then be expressed by Equation 35. In this case, the efficiency at least indicates to what extent the process is able to convert heat into work (or power). The term heat engine (HE) is used to generalize and cover any system producing power from heat.

$$\eta_{HE} \; = \; \frac{W_{cycle}}{Q_H} \tag{35}$$

The thermal efficiencies for heat engines defined by Equations 34 and 35 at least have the advantage of being 0-1 normalized (i.e. values between 0 and 100%). Unfortunately, when considering heat pumps and refrigeration cycles as indicated by the simple illustration in the right part of Figure 9, the thermal efficiencies as defined by Equation 33 are no longer (0-1) normalized. The thermal efficiencies for heat pumps (HP) and refrigeration cycles (RC) are given by Equations 36 and 37 respectively, where a Coefficient of Performance (*COP*) is used rather than η to indicate that the efficiency measures are no longer (0-1) normalized.

$$\eta_{HP} \; = \; COP_{HP} \; = \; \frac{Q_H}{W_{cycle}} \tag{36}$$

$$\eta_{RC} \; = \; COP_{RC} \; = \; \frac{Q_C}{W_{cycle}} \tag{37}$$

For the heat pump, the heat from the cold reservoir is regarded to be "free" and is not included in the thermal efficiency. Similarly, for the refrigeration cycle, the heat delivered to the hot reservoir is not regarded as useful energy, and therefore not included in the thermal efficiency.

The thermal efficiencies provided by Equations 36 and 37 not only could become larger than one; in fact, they should be significantly larger than one to account for the lower Energy Quality of heat compared to power as well as to give payback for the investment cost for the heat pump or refrigeration cycle. As a "rule of thumb", the COP for a heat pump should be at least 3-4 to make its use economically feasible.

5.3 CARNOT EFFICIENCIES

A Carnot cycle is a thermodynamic process with 4 reversible stages, where 2 are adiabatic (thus isentropic) and 2 are isothermal. The *Carnot efficiency* is then the maximum energy efficiency of an ideal process converting heat to power or power to heating or cooling.

If the processes illustrated in Figure 9 are assumed to have negligible heat transfer to/from the surroundings (i.e. there are no losses related to the 1ˢᵗ Law of Thermodynamics), the energy efficiencies in Equations 35-37 can be reformulated as follows:

$$\eta_{HE} = \frac{W_{cycle}}{Q_H} = \frac{Q_H - Q_C}{Q_H} \;\;, \;\; COP_{HP} = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_C} \;\;, \;\; COP_{RC} = \frac{Q_C}{W_{cycle}} = \frac{Q_C}{Q_H - Q_C} \tag{38}$$

Further, if the same processes are assumed to be reversible (i.e. there are no losses related to the 2ⁿᵈ Law of Thermodynamics), the following relation exists between heat duties and temperatures (actually the definition of Kelvin as an absolute temperature scale):

$$\left(\frac{Q_H}{Q_C} \right)_{\substack{int. \\ rev.}} \; = \; \frac{T_H}{T_C} \tag{39}$$

By combining Equations 38 and 39, the following expressions can be derived for the Carnot efficiency for heat engines, heat pumps and refrigeration cycles:

$$\eta_{HE,C} = 1 - \frac{T_C}{T_H} \quad , \quad COP_{HP,C} = \frac{T_H}{T_H - T_C} \quad , \quad COP_{RC,C} = \frac{T_C}{T_H - T_C} \tag{40}$$

If the energy efficiencies of Section 5.2 (that could be referred to as energy conversion factors) are compared with the Carnot efficiencies as references, one can easily derive thermodynamic efficiencies for heat engines, heat pumps and refrigeration cycles.

$$\frac{\eta_E}{\eta_C} = \frac{\text{Energy (thermal) Efficiency}}{\text{Carnot (maximum) Efficiency}} = \eta_{TD} \tag{41}$$

Since the efficiency defined by Equation 41 compares actual behavior with ideal performance, it is by definition a *thermodynamic efficiency* as described in Section 5.1.

Example 2:

To illustrate the use of this "relative" efficiency, consider the refrigeration system in Figure 10, where the temperature of a freezer is maintained at -5°C (268 K).

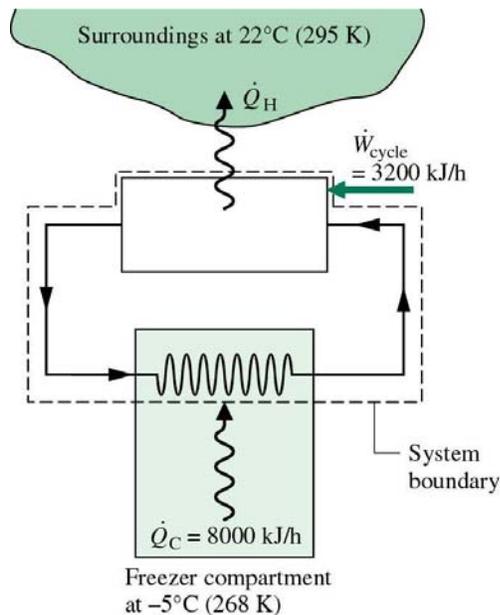Figure 10   Refrigeration Cycle for a Freezer (Moran and Shapiro [1])

The actual performance can be easily evaluated for the refrigeration cycle:

$$COP_{actual} = \frac{\dot{Q}_C}{\dot{W}_{cycle}} = \frac{8000}{3200} = \underline{2.5}$$

The best performance (Carnot cycle) can be established based on the temperatures of the freezer and the environment, while using the expression in Equation 40:

$$COP_{ideal} = \frac{T_C}{T_H - T_C} = \frac{268}{295 - 268} = \underline{9.926}$$

The thermodynamic efficiency can be then found by using Equation 41:

$$\eta_{TD} = \frac{COP_{actual}}{COP_{ideal}} = \frac{2.5}{9.926} = 0.252 = \underline{\underline{25.2\%}}$$

Example 3:

A thorough treatment of the various efficiencies can be found in Skogestad [5], where the interesting example of an electrical heater is discussed. The example also illustrates the important difference between energy efficiency and thermodynamic efficiency. As the name indicates, an electric heater uses electricity (W) to provide heat (Q) to a room where the temperature is maintained at 20ºC. Since electricity can be converted 100% into work, it can be regarded as pure Exergy. Obviously, the electric heater also converts electricity 100% into heat. The energy efficiency for the electric heater is then:

$$\eta_E = \frac{\text{Useful Energy out}}{\text{Useful Energy in}} = \frac{Q}{W} = \frac{W}{W} = 1.0$$

The same amount of electricity could have been used to operate a heat pump between the outdoor temperature $T_C = 5ºC$ (278 K) and the room temperature $T_H = 20ºC$ (293 K). The maximum heat delivered from such a heat pump can be found by using Equation 40:

$$COP_{HP,C} = \frac{T_H}{T_H - T_C} \quad \Rightarrow \quad Q_{max} = W \cdot \frac{T_H}{T_H - T_C} = W \cdot \frac{293}{293 - 278} = 19.533 \cdot W$$

The thermodynamic efficiency for the electric heater is then:

$$\eta_{TD} = \frac{\text{Useful energy out}}{\text{Ideal (max) useful energy out}} = \frac{W}{19.533 \cdot W} = 0.051 = \underline{5.1\%}$$

This very low thermodynamic efficiency is the background for the classical saying that "using electricity for heating of buildings is similar to washing the floor with Champagne".

5.4   EXERGY EFFICIENCIES

Since this document has a focus on Exergy and Energy Quality, the obvious closure of this section on various measures for Energy efficiency, is to derive Exergy efficiencies. Similar to the case with energy (or thermal) efficiency, one can distinguish between total Exergy and useful Exergy being delivered by the system and compare it with the inlet Exergy. While counting total Energy does not make much sense (would give efficiencies very close to 100% in most cases), counting total Exergy could make some sense. An efficiency based on total Exergy would account for thermodynamic losses (referred to as irreversibilities) in the processes, such as the ones caused by heat transfer with $\Delta T > 0$, chemical reaction, mixing, unrestricted expansion, etc.

It is, however, more common to define an Exergy efficiency by counting only <u>useful</u>
Exergy being produced (delivered) by the system. Exergy leaving the system with exhaust
gases, cooling water, etc. are then regarded to be losses as they logically should.

$$\eta_{Ex} \;=\; \frac{\text{Useful Exergy from the Process}}{\text{Total Exergy to the Process}} \tag{42}$$

When applying the *Exergy efficiency* defined by Equation 42 to the case of a heat engine
(for example steam turbine) illustrated in Figure 9 and combining it with Equation 6 that
provides an expression for the Exergy content of heat Q at temperature T, the following
result is obtained (remember that work is pure Exergy):

$$\eta_{Ex} \;=\; \frac{W_{cycle}}{Q_H \cdot \left(1 - \dfrac{T_0}{T_H}\right)} \;=\; \frac{W_{cycle}/Q_H}{\left(1 - \dfrac{T_0}{T_H}\right)} \tag{43}$$

The nominator in Equation 43 is the previously defined energy (or thermal) efficiency.
The denominator <u>would</u> have been equal to the Carnot efficiency provided that the cold
reservoir in Figure 9 had a temperature $T_C = T_0$. For that special case, the following
interesting result is obtained:

$$\eta_{Ex} \;=\; \frac{\eta_E}{\eta_C} \;=\; \eta_{TD} \tag{44}$$

This equality between thermodynamic efficiency and Exergy efficiency requires that the
heat engine uses the surroundings as its cold reservoir. There are of course similarities
between thermodynamic efficiency and Exergy efficiency, since the former measures
actual performance relative to ideal performance, and Exergy is related to maximum work
produced with ideal (reversible) operation. When considering a heat pump, the opposite
exercise, i.e. starting with the thermodynamic efficiency, gives the following result:

$$\eta_{TD} \;=\; \frac{\eta_E}{\eta_C} \;=\; \frac{Q_H/W_{cycle}}{\dfrac{T_H}{T_H - T_C}} \;=\; \frac{Q_H \cdot \left(1 - \dfrac{T_C}{T_H}\right)}{W_{cycle}} \tag{45}$$

Again, if the heat pump uses the surroundings as its cold reservoir (i.e. $T_C = T_0$), the
nominator becomes the Exergy delivered by the system. The denominator is of course the
Exergy input to the system. Thus, a similar result of equality between thermodynamic
efficiency and Exergy efficiency is obtained, identical to the case with the heat engine
(steam turbine):

$$\eta_{TD} \;=\; \frac{\text{Exergy out}}{\text{Exergy in}} \;=\; \eta_{Ex} \tag{46}$$

6.   ENERGY QUALITY

It should be obvious from the previous discussions that different energy forms have different qualities, meaning that they have different abilities to carry out certain tasks. Exergy is the most common measure of Energy Quality and relates to the ability to produce work (see the formal definition in Section 1).

6.1   REVISITING THE EXAMPLE WITH AN ELECTRIC HEATER

The reason why the electric heater in Example 3 had a thermodynamic efficiency of only 5.1% is the fact that high quality energy in the form of electricity (pure Exergy) is converted into low grade heat at only 20ºC. The thermodynamic efficiency of 5.1% was established by considering a heat pump operating between the outdoor temperature of 5ºC and the indoor temperature of 20ºC.

The Exergy efficiency for the same process would be slightly different. Heat at 20ºC has an Exergy content that can be calculated by Equation 6. Notice, however, that since the room temperature of 20ºC is slightly <u>below</u> ambient temperature (25ºC), Equation 6 has to be adjusted as follows:

$$E_x \; = \; Q \cdot \left(\frac{T_0}{T} - 1 \right) \tag{47}$$

The derivation of Equation 47 is equivalent to the derivation of Equation 6 and uses the same analogy to Figure 1.b. For subambient cases, however, the surroundings act as the <u>hot</u> thermal reservoir, while the heat Q at temperature T acts as the <u>cold</u> reservoir. By remembering that for the electric heater, $Q=W$, the Exergy efficiency can be calculated:

$$\eta_{Ex} \; = \; \frac{\text{Exergy out}}{\text{Exergy in}} \; = \; \frac{W \cdot \left(\frac{T_0}{T} - 1 \right)}{W} \; = \; \left(\frac{298}{293} - 1 \right) \; = \; 0.017 \; = \; \underline{1.7\%}$$

Notice that the Exergy efficiency in this case is even lower than the thermodynamic efficiency. The reason is that Exergy relates the room temperature (20ºC) to ambient temperature (25ºC), while the thermodynamic efficiency was established by relating the room temperature to the outdoor temperature (5ºC).

6.2   EXERGY OF VARIOUS ENERGY FORMS

The Energy Quality (or Exergy) of *thermal energy* (or heat) has been explained in large detail. Figure 11 shows a graph of the Carnot efficiency (reflecting Exergy) of heat as a function of temperature using Equation 6 above ambient and Equation 47 below ambient.

When examining Equation 47, the Carnot efficiency of heat actually becomes larger than 100% for temperatures below half of the ambient temperature (expressed in Kelvin):

$$\frac{T_0}{T} > 2 \quad \Rightarrow \quad T < 0.5 \cdot T_0 \quad \Rightarrow \quad T < 149 \text{ K} \quad \text{or} \quad T < -124 \text{ °C}$$

This somewhat surprising result means that below -124ºC, heat (or actually a cold resource) is more valuable than work. This fact is well known and actually utilized when designing cryogenic processes such as LNG and Hydrogen liquefaction processes.
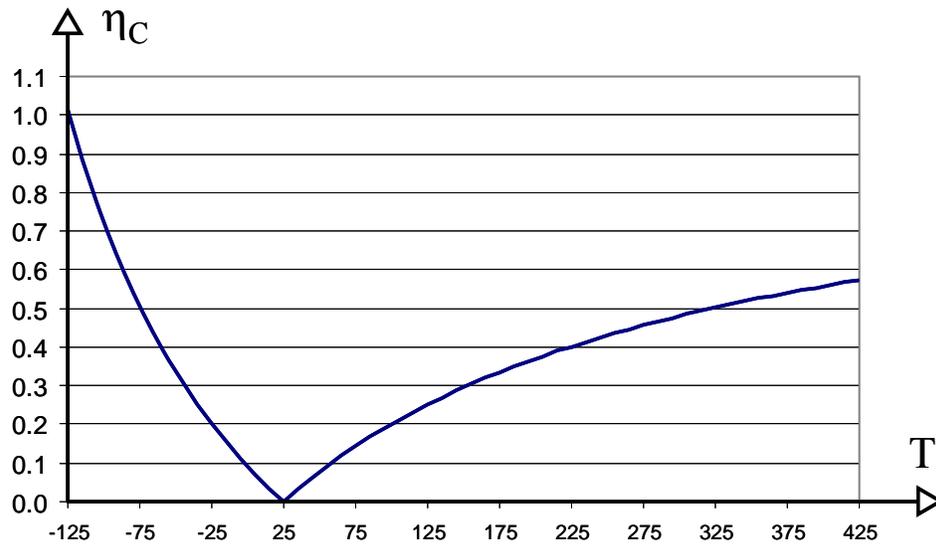

Figure 11   The Exergy of Heat (or Carnot factor) as a function of Temperature.

Since Energy Quality in the form of Exergy is counted as the ability to produce work, it is obvious that *electricity* has the ability to completely be converted into work. In practice, however, there are some minor losses when electricity is used for example to drive an electric motor. Closely related is *mechanical energy* that can be produced by gas and steam turbines. This energy form is also pure Exergy in the sense that these turbines can be used to drive compressors and pumps directly (i.e. on the same shaft) without converting to electricity by the use of a generator.

Even *chemical energy* is in principle pure Exergy, at least if it could be converted to electricity without any temperature change (entropy production) in a fuel cell. In practice, chemical energy (say in fossil fuels) is utilized in combustion to produce steam or run a gas turbine. There are large exergy losses related to combustion reactions, and in a steam boiler, there are also considerable exergy losses related to heat transfer at large temperature driving forces between the exhaust gas and the steam system.

6.3   EXERGY AND INDUSTRIAL ECOLOGY

Industrial Ecology is a systems oriented scientific discipline with a strong focus on environmental impact of human activities. Within Industrial Ecology, Life Cycle Assessment (LCA) is a well known analysis methodology trying to capture the entire life span of a product from "cradle" to "grave".

When dealing with materials, energy forms and sustainability, the Industrial Ecology community [6] uses Exergy as a measure of quality to answer questions such as:

- Does the human production of one unit of an economic good by method *A* utilize more of a resource's exergy than by method *B*?

- Does the human production of an economic good deplete the exergy of the Earth's natural resources more quickly than those resources are able to receive exergy?

7. EXERGY ANALYSIS

Exergy Analysis has been extensively used in some areas, more moderately used in other areas, to evaluate the energy efficiency of power stations, industrial plants and process equipment. The methodology should be used with care, however, since there are a number of pitfalls and limitations. First, Exergy Analysis is better suited for analyzing process equipment (the unit operations level) than total processes (the systems level). Secondly, the link between Exergy and cost is weak at best; in fact there is often a conflict between reducing Exergy losses and reducing investment cost.

7.1 EXERGY ANALYSIS OF HEAT EXCHANGERS

Two examples will be considered in this Section. In the first case (Figure 12.a) heat is transferred from a condensing hot stream to a vaporizing cold stream. Assuming that both streams contain only a single chemical component, phase change takes place at constant temperature. In the second case (Figure 12.b); heat is transferred from a hot stream to a cold stream, where both streams are subject to a change in sensible heat only, with no phase changes. For the first case, Equation 6 can be used to calculate Exergy changes for the two participating streams. For the second case, however, new relations have to be derived for the case of non-constant temperatures.
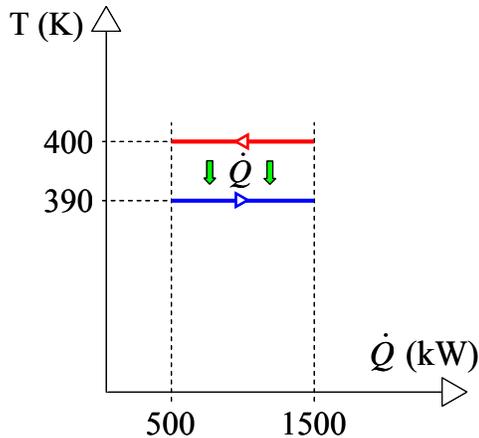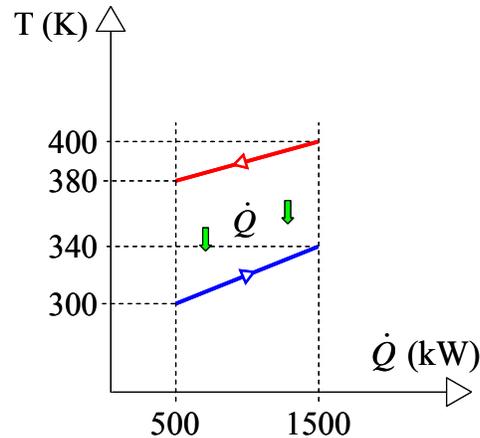

Figure 12.a   Heat transfer between streams   Figure 12.b   Heat transfer between streams
at constant temperature   with gliding temperatures

For the first case of heat transfer at constant temperatures, Equation 6 can be used directly to calculate the Exergy loss of the hot stream at 400 K and the Exergy gain of the cold stream at 390 K when $\dot{Q}$ as indicated in Figure 12.b is 1000 kW. Notice that the normal sign convention is <u>not</u> used here, thus the heat transfer per time $\dot{Q}$ is regarded as a positive entity for both the hot and the cold stream. The total change in Exergy (here Exergy losses) due to heat transfer at $\Delta T > 0$ is then the following:

$$\Delta \dot{E}_x \;=\; \dot{m}_H \cdot \Delta e_{x,H} + \dot{m}_C \cdot \Delta e_{x,C} \;=\; -\dot{Q} \cdot \left(1 - \frac{T_0}{T_H}\right) + \dot{Q} \cdot \left(1 - \frac{T_0}{T_C}\right)$$

$$= \; \dot{Q} \cdot T_0 \cdot \left(\frac{1}{T_H} - \frac{1}{T_C}\right) \;=\; 1000 \cdot 298 \cdot \left(\frac{1}{400} - \frac{1}{390}\right) \;=\; \underline{-19.1\ \text{kW}}$$

For the second case of heat transfer with changing temperatures, a new formulation has to be established starting with a differential form of Equation 6. For an infinitesimal amount of heat δQ extracted from a hot stream at temperature T_H, the inherent change in Exergy is given by Equation 48. It should be obvious that a stream giving away heat reduces its ability to produce work, thus its Exergy is reduced, which is why there is a negative sign in Equation 48. Correspondingly, if heat is supplied to a cold stream at temperature T_C, the inherent change in Exergy is given by Equation 49. It should be noted that the normal sign convention in thermodynamics is <u>not</u> used here, thus the infinitesimal amount of heat δQ is regarded as a <u>positive</u> entity for both hot (Equation 48) and cold streams (Equation 49).

$$dE_{x,H} \;=\; -\left(1 - \frac{T_0}{T_H}\right) \cdot \delta Q \qquad\qquad (48)$$

$$dE_{x,C} \;=\; \left(1 - \frac{T_0}{T_C}\right) \cdot \delta Q \qquad\qquad (49)$$

In Figure 12.b, the specific heat capacities of the two streams are assumed to be constant, resulting in linear temperature-enthalpy relations. Integration of Equations 48 and 49 with varying T_H and T_C then becomes a simple dT/T integration which results in logarithmic expressions. The total change in Exergy (here Exergy losses) due to heat transfer is then:

$$\Delta \dot{E}_x \;=\; \dot{m}_H \cdot \Delta e_{x,H} + \dot{m}_C \cdot \Delta e_{x,C} \;=\; \dot{Q} \cdot T_0 \cdot \left(\frac{1}{T_{H,LM}} - \frac{1}{T_{C,LM}}\right) \qquad (50)$$

In Equation 50, the logarithmic mean temperatures for the hot and cold streams are defined and can be calculated for the case in Figure 12.b as follows:

$$T_{H,LM} \;=\; \frac{T_{H,in} - T_{H,out}}{\ln \dfrac{T_{H,in}}{T_{H,out}}} \;=\; \frac{400 - 380}{\ln \dfrac{400}{380}} \;=\; 389.92\ \text{K}$$

$$T_{C,LM} \;=\; \frac{T_{C,out} - T_{C,in}}{\ln \dfrac{T_{C,out}}{T_{C,in}}} \;=\; \frac{340 - 300}{\ln \dfrac{340}{300}} \;=\; 319.58\ \text{K}$$

Equation 50 can now be used to calculate the total change in Exergy (here Exergy losses) due to heat transfer for the second case with changing temperatures:

$$\Delta \dot{E}_x \;=\; \dot{Q} \cdot T_0 \cdot \left(\frac{1}{T_{H,LM}} - \frac{1}{T_{C,LM}}\right) \;=\; 1000 \cdot 298 \cdot \left(\frac{1}{389.92} - \frac{1}{319.58}\right) \;=\; \underline{-168.2\ \text{kW}}$$

NOMENCLATURE

Symbols

B	Total exergy function (in kJ)
c_p	Specific heat capacity at constant pressure (in kJ/kg·K)
c_v	Specific heat capacity at constant volume (in kJ/kg·K)
d	Differential operator
E	Energy (in kJ or kWh)
E	Exergy (in kJ or kWh) when used with subscript x
e	Specific exergy (in kJ/kg) when used with subscript x
G	Gibbs free energy (kJ or kWh)
g	Acceleration of gravity (9.81 m/s^2)
H	Total enthalpy (in kJ or kWh)
h	Specific enthalpy (in kJ/kg)
k	Ratio between specific heat capacities
M	Molecular weight (or mass) (in kg/kmol)
$\dot{m}$	Mass flowrate (in kg/s)
p	Pressure of a system (in bar)
Q	Heat (in kJ or kWh)
$\dot{Q}$	Heat flow (in kJ/s or kW)
R	Mass based (component dependent) gas constant (in kJ/kg·K)
$\bar{R}$	Universal gas constant (8.314 kJ/kmole·K)
S	Total entropy (in kJ/K)
s	Specific entropy (in kJ/kg·K)
T	Temperature of a system (in °C or K)
t	Time (in s)
U	Internal energy (in kJ or kWh)
V	Linear velocity (in m/s)
V	Total volume of a system (in m^3)
W	Work (in kJ or kWh)
$\dot{W}$	Power (or work per time) (in kW)
z	Horizontal position (altitude) (in m)

Greek Symbols

β	Specific exergy function (in kJ/kg)
Δ	Change in property
δ	Small (infinitesimal) amount
η	Efficiency
$\dot{\sigma}$	Entropy production (in kJ/K·s or kW/K)

Subscripts

0	Ambient conditions (pressure and temperature)
C	Cold stream or reservoir
c	Carnot or compressor
COP	Coefficient of Performance
cv	Control volume

cycle	Thermodynamic cyclic process
E	Energy
e	Outlet (or exit) (streams)
Ex	Exergy
H	Hot stream or reservoir
HE	Heat Engine
HP	Heat Pump
i	Inlet (streams)
in	Inlet
int.rev.	Internally reversible
j	Heat flow
k	Kinetic energy
LM	Logarithmic mean
max	Maximum
out	Outlet
p	Potential energy
RC	Refrigeration Cycle
s	Isentropic
t	Turbine
TD	Thermodynamic
x	Exergy

Superscripts

p	Pressure (part of thermo-mechanical exergy)
T	Temperature (part of thermo-mechanical exergy)
tm	Thermo-mechanical (exergy)

REFERENCES

[1] Moran M.J. and Shapiro H.N., *Fundamentals of Engineering Thermodynamics*, John Wiley & Sons, 5th edition (SI Units), 2006.

[2] Rant Z., Exergy, a new word for "technical available work", *Forschung auf dem Gebiete des Ingenieurwesens* (in German), vol. 22, pp. 36-37, 1956.

[3] Szargut J., International progress in second law analysis, *Energy*, vol. 5, no. 8-9, pp. 709-718, 1980.

[4] Kotas T.J., *The exergy method of thermal plant analysis*, Krieger Publishing Company, Malabar, Florida, 1995.

[5] Skogestad S., *Chemical and Energy Process Engineering*, CRC Press, Boca Raton, Florida, 2009.

[6] Wikipedia, http://en.wikipedia.org/wiki/Exergy , visited 8 November 2008.